Lesson 4: Calculating Probabilities for Chance Experiments with Equally Likely Outcomes

Classwork

Examples: Theoretical Probability

In a previous lesson, you saw that to find an estimate of the probability of an event for a chance experiment you divide

$$P(\text{event}) = \frac{\text{Number of observed occurrences of the event}}{\text{Total number of observations}}.$$

Your teacher has a bag with some cubes colored yellow, green, blue, and red. The cubes are identical except for their color. Your teacher will conduct a chance experiment by randomly drawing a cube with replacement from the bag. Record the outcome of each draw in the table below.

Trial	Outcome
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

1. Based on the 20 trials, estimate for the probability of
   a. choosing a yellow cube.

   b. choosing a green cube.

   c. choosing a red cube.

   d. choosing a blue cube.

2. If there are 40 cubes in the bag, how many cubes of each color are in the bag? Explain.

3. If your teacher were to randomly draw another 20 cubes one at a time and with replacement from the bag, would you see exactly the same results? Explain.

4. Find the fraction of each color of cubes in the bag.

   Yellow

   Green

   Red

   Blue

Each fraction is the *theoretical probability* of choosing a particular color of cube when a cube is randomly drawn from the bag.

When all the possible outcomes of an experiment are equally likely, the probability of each outcome is

$$P(\text{outcome}) = \frac{1}{\text{Number of possible outcomes}}.$$

An event is a collection of outcomes, and when the outcomes are equally likely, the theoretical probability of an event can be expressed as

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}.$$

The theoretical probability of drawing a blue cube is

$$P(\text{blue}) = \frac{\text{Number of blue cubes}}{\text{Total number of cubes}} = \frac{10}{40}.$$

5. Is each color equally likely to be chosen? Explain your answer.

6. How do the theoretical probabilities of choosing each color from Exercise 4 compare to the experimental probabilities you found in Exercise 1?

7. An experiment consisted of flipping a nickel and a dime. The first step in finding the theoretical probability of obtaining a heads on the nickel and a heads on the dime is to list the sample space. For this experiment, the sample space is shown below.

Nickel	Dime
H	H
H	T
T	H
T	T

If the counts are fair, these outcomes are equally likely, so the probability of each outcome is $\frac{1}{4}$.

Nickel	Dime	Probability
H	H	$\frac{1}{4}$
H	T	$\frac{1}{4}$
T	H	$\frac{1}{4}$
T	T	$\frac{1}{4}$

The probability of two heads is $\frac{1}{4}$ or $P(\text{two heads}) = \frac{1}{4}$.

Exercises

1. Consider a chance experiment of rolling a number cube.
   a. What is the sample space? List the probability of each outcome in the sample space.

   b. What is the probability of rolling an odd number?

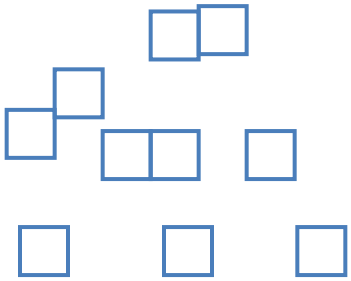
   c. What is the probability of rolling a number less than 5?

2. Consider an experiment of randomly selecting a letter from the word *number*.
   a. What is the sample space? List the probability of each outcome in the sample space.

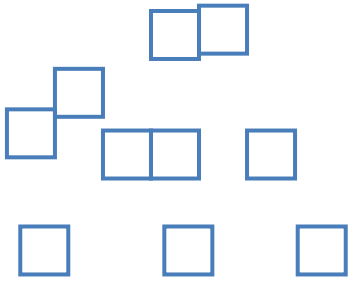
   b. What is the probability of selecting a vowel?

   c. What is the probability of selecting the letter z?

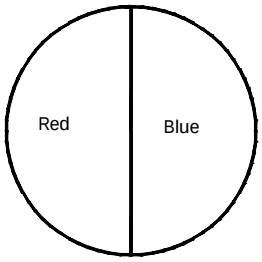
3. Consider an experiment of randomly selecting a cube from a bag of 10 cubes.
   a. Color the cubes below so that the probability of selecting a blue cube is $\frac{1}{2}$.

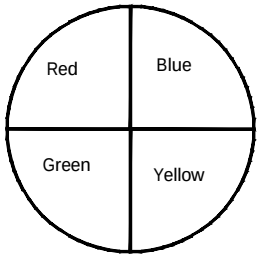


b. Color the cubes below so that the probability of selecting a blue cube is $\frac{4}{5}$.



4. Students are playing a game that requires spinning the two spinners shown below. A student wins the game if both spins land on red. What is the probability of winning the game? Remember to first list the sample space and the probability of each outcome in the sample space. There are eight possible outcomes to this chance experiment.





Lesson Summary

When all the possible outcomes of an experiment are equally likely, the probability of each outcome is

$$P(\text{outcome}) = \frac{1}{\text{Number of possible outcomes}}.$$

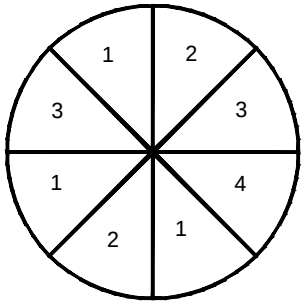
An event is a collection of outcomes, and when all outcomes are equally likely, the theoretical probability of an event can be expressed as

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}.$$

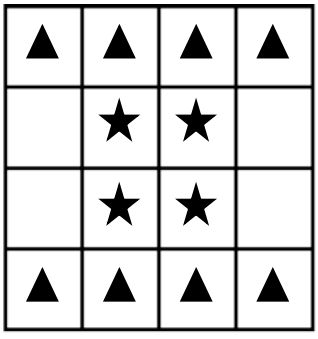
Problem Set

1. In a seventh grade class of 28 students, there are 16 girls and 12 boys. If one student is randomly chosen to win a prize, what is the probability that a girl is chosen?

2. An experiment consists of spinning the spinner once.
   a. Find the probability of landing on a 2.
   b. Find the probability of landing on a 1.
   c. Is landing in each section of the spinner equally likely to occur? Explain.



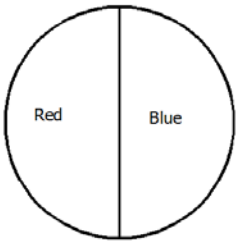
3. An experiment consists of randomly picking a square section from the board shown below.
   a. Find the probability of choosing a triangle.
   b. Find the probability of choosing a star.
   c. Find the probability of choosing an empty square.
   d. Find the probability of choosing a circle.

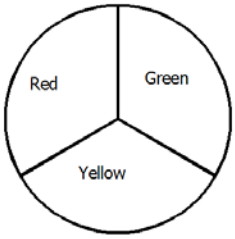


4. Seventh graders are playing a game where they randomly select two integers from 0–9, inclusive, to form a two-digit number. The same integer might be selected twice.
   a. List the sample space for this chance experiment. List the probability of each outcome in the sample space.
   b. What is the probability that the number formed is between 90 and 99, inclusive?
   c. What is the probability that the number formed is evenly divisible by 5?
   d. What is the probability that the number formed is a factor of 64?

5. A chance experiment consists of flipping a coin and rolling a number cube with the numbers 1–6 on the faces of the cube.
   a. List the sample space of this chance experiment. List the probability of each outcome in the sample space.
   b. What is the probability of getting a heads on the coin and the number 3 on the number cube?
   c. What is the probability of getting a tails on the coin and an even number on the number cube?

6. A chance experiment consists of spinning the two spinners below.





   a. List the sample space and the probability of each outcome.
   b. Find the probability of the event of getting a red on the first spinner and a red on the second spinner.
   c. Find the probability of a red on at least one of the spinners.