## Lesson 6: Using Tree Diagrams to Represent a Sample Space and

 to Calculate Probabilities
## Classwork

Suppose a girl attends a preschool where the students are studying primary colors. To help teach calendar skills, the teacher has each student maintain a calendar in his or her cubby. For each of the four days that they are covering primary colors in class, students get to place a colored dot on their calendar: blue, yellow, or red. When the four days of the school week have passed (Monday-Thursday), what might the young girl's calendar look like?

One outcome would be four blue dots if the student chose blue each day. But consider that the first day (Monday) could be blue, and the next day (Tuesday) could be yellow, and Wednesday could be blue, and Thursday could be red. Or, maybe Monday and Tuesday could be yellow, Wednesday could be blue, and Thursday could be red. Or, maybe Monday, Tuesday, and Wednesday could be blue, and Thursday could be red, and so on and so forth.

As hard to follow as this seems now, we have only mentioned 3 of the 81 possible outcomes in terms of the four days of colors! Listing the other 78 outcomes would take several pages! Rather than listing outcomes in the manner described above (particularly when the situation has multiple stages, such as the multiple days in the case above), we often use a tree diagram to display all possible outcomes visually. Additionally, when the outcomes of each stage are the result of a chance experiment, tree diagrams are helpful for computing probabilities.

## Example 1: Two Nights of Games

Imagine that a family decides to play a game each night. They all agree to use a tetrahedral die (i.e., a four-sided pyramidal die where each of four possible outcomes is equally likely-see image on page S.44) each night to randomly determine if they will play a board game (B) or a card game (C). The tree diagram mapping the possible overall outcomes over two consecutive nights will be developed below.

To make a tree diagram, first present all possibilities for the first stage. (In this case, Monday.)

## Monday Tuesday Outcome

B

C

Then, from each branch of the first stage, attach all possibilities for the second stage (Tuesday).
Monday Tuesday Outcome


Note: If the situation has more than two stages, this process would be repeated until all stages have been presented.
a. If BB represents two straight nights of board games, what does CB represent?
b. List the outcomes where exactly one board game is played over two days. How many outcomes were there?

## Example 2: Two Nights of Games (with Probabilities)

In the example above, each night's outcome is the result of a chance experiment (rolling the tetrahedral die). Thus, there is a probability associated with each night's outcome.

By multiplying the probabilities of the outcomes from each stage, we can obtain the probability for each "branch of the tree." In this case, we can figure out the probability of each of our four outcomes: BB, BC, CB, and CC.

For this family, a card game will be played if the die lands showing a value of 1 , and a board game will be played if the die lands showing a value of 2,3 , or 4 . This makes the probability of a board game (B) on a given night 0.75 .

a. The probabilities for two of the four outcomes are shown. Now, compute the probabilities for the two remaining outcomes.
b. What is the probability that there will be exactly one night of board games over the two nights?

## Exercises: Two Children

Two friends meet at a grocery store and remark that a neighboring family just welcomed their second child. It turns out that both children in this family are girls, and they are not twins. One of the friends is curious about what the chances are of having 2 girls in a family's first 2 births. Suppose that for each birth the probability of a boy birth is 0.5 and the probability of a girl birth is also 0.5 .

1. Draw a tree diagram demonstrating the four possible birth outcomes for a family with 2 children (no twins). Use the symbol B for the outcome of boy and G for the outcome of girl. Consider the first birth to be the first stage. (Refer to Example 1 if you need help getting started.)
2. Write in the probabilities of each stage's outcome to the tree diagram you developed above, and determine the probabilities for each of the 4 possible birth outcomes for a family with 2 children (no twins).
3. What is the probability of a family having 2 girls in this situation? Is that greater than or less than the probability of having exactly 1 girl in 2 births?

## Lesson Summary

Tree diagrams can be used to organize outcomes in the sample space for chance experiments that can be thought of as being performed in multiple stages. Tree diagrams are also useful for computing probabilities of events with more than one outcome.

## Problem Set

1. Imagine that a family of three (Alice, Bill, and Chester) plays bingo at home every night. Each night, the chance that any one of the three players will win is $\frac{1}{3}$.
a. Using A for Alice wins, B for Bill wins, and C for Chester wins, develop a tree diagram that shows the nine possible outcomes for two consecutive nights of play.
b. Is the probability that "Bill wins both nights" the same as the probability that "Alice wins the first night and Chester wins the second night"? Explain.
2. According to the Washington, D.C. Lottery's website for its Cherry Blossom Doubler instant scratch game, the chance of winning a prize on a given ticket is about $17 \%$. Imagine that a person stops at a convenience store on the way home from work every Monday and Tuesday to buy a Scratcher ticket to play the game.
(Source: http://dclottery.com/games/scratchers/1223/cherry-blossom-doubler.aspx accessed May 27, 2013).
a. Develop a tree diagram showing the four possible outcomes of playing over these two days. Call stage 1 "Monday," and use the symbols W for a winning ticket and L for a non-winning ticket.
b. What is the chance that the player will not win on Monday but will win on Tuesday?
c. What is the chance that the player will win at least once during the two-day period?

## Image of Tetrahedral Die

Source: http://commons.wikimedia.org/wiki/File:4-sided dice 250.jpg
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