## Lesson 7

## Lesson 7: Calculating Probabilities of Compound Events

## Classwork

A previous lesson introduced tree diagrams as an effective method of displaying the possible outcomes of certain multistage chance experiments. Additionally, in such situations, tree diagrams were shown to be helpful for computing probabilities.

In those previous examples, diagrams primarily focused on cases with two stages. However, the basic principles of tree diagrams can apply to situations with more than two stages.

## Example 1: Three Nights of Games

Recall a previous example where a family decides to play a game each night, and they all agree to use a tetrahedral die (a four-sided die in the shape of a pyramid where each of four possible outcomes is equally likely) each night to randomly determine if the game will be a board (B) or a card (C) game. The tree diagram mapping the possible overall outcomes over two consecutive nights was as follows:


But how would the diagram change if you were interested in mapping the possible overall outcomes over three consecutive nights? To accommodate this additional third stage, you would take steps similar to what you did before. You would attach all possibilities for the third stage (Wednesday) to each branch of the previous stage (Tuesday).


## Exercises 1-3

1. If BBB represents three straight nights of board games, what does CBB represent?
2. List all outcomes where exactly two board games were played over three days. How many outcomes were there?
3. There are eight possible outcomes representing the three nights. Are the eight outcomes representing the three nights equally likely? Why or why not?

## Example 2: Three Nights of Games (with Probabilities)

In the example above, each night's outcome is the result of a chance experiment (rolling the four-sided die). Thus, there is a probability associated with each night's outcome.

By multiplying the probabilities of the outcomes from each stage, you can obtain the probability for each "branch of the tree." In this case, you can figure out the probability of each of our eight outcomes.

For this family, a card game will be played if the die lands showing a value of 1 , and a board game will be played if the die lands showing a value of 2,3 , or 4 . This makes the probability of a board game (B) on a given night 0.75 .

Let's use a tree to examine the probabilities of the outcomes for the three days.


## Exercises 4-6

4. Probabilities for two of the eight outcomes are shown. Calculate the approximate probabilities for the remaining six outcomes.
5. What is the probability that there will be exactly two nights of board games over the three nights?
6. What is the probability that the family will play at least one night of card games?

## Exercises 7-10: Three Children

A neighboring family just welcomed their third child. It turns out that all 3 of the children in this family are girls, and they are not twins or triplets. Suppose that for each birth the probability of a boy birth is 0.5 and the probability of a girl birth is also 0.5 . What are the chances of having 3 girls in a family's first 3 births?
7. Draw a tree diagram showing the eight possible birth outcomes for a family with 3 children (no twins or triplets). Use the symbol B for the outcome of boy and G for the outcome of girl. Consider the first birth to be the first stage. (Refer to Example 1 if you need help getting started.)
8. Write in the probabilities of each stage's outcomes in the tree diagram you developed above, and determine the probabilities for each of the eight possible birth outcomes for a family with 3 children (no twins).

## Lesson 7

9. What is the probability of a family having 3 girls in this situation? Is that greater than or less than the probability of having exactly 2 girls in 3 births?
10. What is the probability of a family of 3 children having at least 1 girl?

## Lesson Summary

The use of tree diagrams is not limited to cases of just two stages. For more complicated experiments, tree diagrams are used to organize outcomes and to assign probabilities. The tree diagram is a visual representation of outcomes that involve more than one event.

## Problem Set

1. According to the Washington, D.C. Lottery's website for its Cherry Blossom Double" instant scratch game, the chance of winning a prize on a given ticket is about $17 \%$. Imagine that a person stops at a convenience store on the way home from work every Monday, Tuesday, and Wednesday to buy a Scratcher ticket and plays the game.
(Source: http://dclottery.com/games/scratchers/1223/cherry-blossom-doubler.aspx accessed May 27, 2013)
a. Develop a tree diagram showing the eight possible outcomes of playing over these three days. Call stage one "Monday," and use the symbols W for a winning ticket and L for a non-winning ticket.
b. What is the probability that the player will not win on Monday but will win on Tuesday and Wednesday?
c. What is the probability that the player will win at least once during the 3-day period?
2. A survey company is interested in conducting a statewide poll prior to an upcoming election. They are only interested in talking to registered voters.

Imagine that 55\% of the registered voters in the state are male and $45 \%$ are female. Also, consider that the distribution of ages may be different for each group. In this state, $30 \%$ of male registered voters are age 18-24, $37 \%$ are age $25-64$, and $33 \%$ are 65 or older. $32 \%$ of female registered voters are age $18-24,26 \%$ are age $25-64$, and $42 \%$ are 65 or older.

The following tree diagram describes the distribution of registered voters. The probability of selecting a male registered voter age $18-24$ is 0.165 .

a. What is the chance that the polling company will select a registered female voter age 65 or older?
b. What is the chance that the polling company will select any registered voter age 18-24?

