

## Lesson 11: Conducting a Simulation to Estimate the Probability of an Event

### Classwork

#### Examples 1–2: Simulation

In the last lesson, we used coins, number cubes, and cards to carry out simulations. Another option is putting identical pieces of paper or colored disks into a container, mixing them thoroughly, and then choosing one.

For example, if a basketball player typically makes five out of eight foul shots, then a colored disk could be used to simulate a foul shot. A green disk could represent a made shot, and a red disk could represent a miss. You could put five green and three red disks in a container, mix them, and then choose one to represent a foul shot. If the color of the disk is green, then the shot is made. If the color of the disk is red, then the shot is missed. This procedure simulates one foul shot.

1. Using colored disks, describe how one at-bat could be simulated for a baseball player who has a batting average of 0.300. Note that a batting average of 0.300 means the player gets a hit (on average) three times out of every ten times at bat. Be sure to state clearly what a color represents.
2. Using colored disks, describe how one at-bat could be simulated for a player who has a batting average of 0.273. Note that a batting average of 0.273 means that on average, the player gets 273 hits out of 1,000 at-bats.

**Example 3: Using Random Number Tables**

Why is using colored disks not practical for the situation described in Example 2? Another way to carry out a simulation is to use a random number table, or a random number generator. In a random number table, the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 occur equally often in the long run. Pages and pages of random numbers can be found online.

For example, here are three lines of random numbers. The space after every five digits is only for ease of reading. Ignore the spaces when using the table.

25256 65205 72597 00562 12683 90674 78923 96568 32177 33855  
76635 92290 88864 72794 14333 79019 05943 77510 74051 87238  
07895 86481 94036 12749 24005 80718 13144 66934 54730 77140

To use the random number table to simulate an at-bat for the 0.273 hitter in Exercise 2, you could use a three-digit number to represent one at bat. The three-digit numbers from 000–272 could represent a hit, and the three-digit numbers from 273–999 could represent a non-hit. Using the random numbers above and starting at the beginning of the first line, the first three-digit random number is 252, which is between 000 and 272, so that simulated at-bat is a hit. The next three-digit random number is 566, which is a non-hit.

Continuing on the first line of the random numbers above, what would the hit/non-hit outcomes be for the next six at-bats? Be sure to state the random number and whether it simulates a hit or non-hit.

**Example 4: Baseball Player**

A batter typically gets to bat four times in a ballgame. Consider the 0.273 hitter from the previous example. Use the following steps (and the random numbers shown above) to estimate that player's probability of getting at least three hits (three or four) in four times at-bat.

- Describe what one trial is for this problem.
- Describe when a trial is called a success and when it is called a failure.
- Simulate 12 trials. (Continue to work as a class, or let students work with a partner.)

- d. Use the results of the simulation to estimate the probability that a 0.273 hitter gets three or four hits in four times at-bat. Compare your estimate with other groups.

### Example 5: Birth Month

In a group of more than 12 people, is it likely that at least two people, maybe more, will have the same birth month? Why? Try it in your class.

Now, suppose that the same question is asked for a group of only seven people. Are you likely to find some groups of seven people in which there is a match but other groups in which all seven people have different birth months? In the following exercise, you will estimate the probability that at least two people in a group of seven were born in the same month.

### Exercises 1–4

1. What might be a good way to generate outcomes for the birth month problem—using coins, number cubes, cards, spinners, colored disks, or random numbers?
2. How would you simulate one trial of seven birth months?



### Lesson Summary

In the previous lesson, you carried out simulations to estimate a probability. In this lesson, you had to provide parts of a simulation design. You also learned how random numbers could be used to carry out a simulation.

To design a simulation:

- Identify the possible outcomes, and decide how to simulate them, using coins, number cubes, cards, spinners, colored disks, or random numbers.
- Specify what a trial for the simulation will look like and what a success and a failure would mean.
- Make sure you carry out enough trials to ensure that the estimated probability gets closer to the actual probability as you do more trials. There is no need for a specific number of trials at this time; however, you want to make sure to carry out enough trials so that the relative frequencies level off.

### Problem Set

1. A model airplane has two engines. It can fly if one engine fails but is in serious trouble if both engines fail. The engines function independently of one another. On any given flight, the probability of a failure is 0.10 for each engine. Design a simulation to estimate the probability that the airplane will be in serious trouble the next time it goes up.
  - a. How would you simulate the status of an engine?
  - b. What constitutes a trial for this simulation?
  - c. What constitutes a success for this simulation?
  - d. Carry out 50 trials of your simulation, list your results, and calculate an estimate of the probability that the airplane will be in serious trouble the next time it goes up.
2. In an effort to increase sales, a cereal manufacturer created a really neat toy that has six parts to it. One part is put into each box of cereal. Which part is in a box is not known until the box is opened. You can play with the toy without having all six parts, but it is better to have the complete set. If you are really lucky, you might only need to buy six boxes to get a complete set. But if you are very unlucky, you might need to buy many, many boxes before obtaining all six parts.
  - a. How would you represent the outcome of purchasing a box of cereal, keeping in mind that there are six different parts? There is one part in each box.
  - b. What constitutes a trial in this problem?
  - c. What constitutes a success in a trial in this problem?
  - d. Carry out 15 trials, list your results, and compute an estimate of the probability that it takes the purchase of 10 or more boxes to get all six parts.

3. Suppose that a type A blood donor is needed for a certain surgery. Carry out a simulation to answer the following question: If 40% of donors have type A blood, what is an estimate of the probability that it will take at least four donors to find one with type A blood?
- How would you simulate a blood donor having or not having type A?
  - What constitutes a trial for this simulation?
  - What constitutes a success for this simulation?
  - Carry out 15 trials, list your results, and compute an estimate for the probability that it takes at least four donors to find one with type A blood.