Lesson 12: Applying Probability to Make Informed Decisions

Classwork

Example 1: Number Cube

Your teacher gives you a number cube with numbers 1–6 on its faces. You have never seen that particular cube before. You are asked to state a theoretical probability model for rolling it once. A probability model consists of the list of possible outcomes (the sample space) and the theoretical probabilities associated with each of the outcomes. You say that the probability model might assign a probability of $\frac{1}{6}$ to each of the possible outcomes, but because you have never seen this particular cube before, you would like to roll it a few times. (Maybe it is a trick cube.) Suppose your teacher allows you to roll it 500 times, and you get the following results:

Outcome	1	2	3	4	5	6
Frequency	77	92	75	90	76	90

Exercises 1–2

1. If the equally likely model were correct, about how many of each outcome would you expect to see if the cube is rolled 500 times?

2. Based on the data from the 500 rolls, how often were odd numbers observed? How often were even numbers observed?

Example 2: Probability Model

Two black balls and two white balls are put in a small cup whose bottom allows the four balls to fit snugly. After shaking the cup well, two patterns of colors are possible as shown. The pattern on the left shows the similar colors are opposite each other, and the pattern on the right shows the similar colors are next to or adjacent to each other.
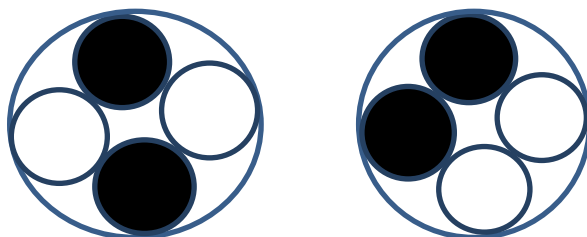
Philippe is asked to specify a probability model for the chance experiment of shaking the cup and observing the pattern. He thinks that because there are two outcomes—like heads and tails on a coin—that the outcomes should be equally likely. Sylvia isn't so sure that the equally likely model is correct, so she would like to collect some data before deciding on a model.

Exercise 3

3. Collect data for Sylvia. Carry out the experiment of shaking a cup that contains four balls, two black and two white, observing, and recording whether the pattern is opposite or adjacent. Repeat this process 20 times. Then, combine the data with that collected by your classmates.

    Do your results agree with Philippe's equally likely model, or do they indicate that Sylvia had the right idea? Explain.

Exercises 4–5

There are three popular brands of mixed nuts. Your teacher loves cashews, and in his experience of having purchased these brands, he suggests that not all brands have the same percentage of cashews. One has around 20% cashews, one has 25%, and one has 35%.

Your teacher has bags labeled A, B, and C representing the three brands. The bags contain red beads representing cashews and brown beads representing other types of nuts. One bag contains 20% red beads, another 25% red beads, and the third has 35% red beads. You are to determine which bag contains which percentage of cashews. You cannot just open the bags and count the beads.

4. Work as a class to design a simulation. You need to agree on what an outcome is, what a trial is, what a success is, and how to calculate the estimated probability of getting a cashew. Base your estimate on 50 trials.

5. Your teacher will give your group one of the bags labeled A, B, or C. Using your plan from part (a), collect your data. Do you think you have the 20%, 25%, or 35% cashews bag? Explain.

Exercises 6–8

Suppose you have two bags, A and B, in which there are an equal number of slips of paper. Positive numbers are written on the slips. The numbers are not known, but they are whole numbers between 1 and 75, inclusive. The same number may occur on more than one slip of paper in a bag.

These bags are used to play a game. In this game, you choose one of the bags, and then choose one slip from that bag. If you choose bag A, and the number you choose from it is a prime number, then you win. If you choose bag B, and the number you choose from it is a power of 2, you win. Which bag should you choose?

6. Emma suggests that it doesn't matter which bag you choose because you don't know anything about what numbers are inside the bags. So she thinks that you are equally likely to win with either bag. Do you agree with her? Explain.

7. Aamir suggests that he would like to collect some data from both bags before making a decision about whether or not the model is equally likely. Help Aamir by drawing 50 slips from each bag, being sure to replace each one before choosing again. Each time you draw a slip, record whether it would have been a winner or not. Using the results, what is your estimate for the probability of drawing a prime number from bag A and drawing a power of 2 from bag B?

8. If you were to play this game, which bag would you choose? Explain why you would pick this bag.

Lesson Summary

This lesson involved knowing the probabilities for outcomes of a sample space. You used these probabilities to determine whether or not the simulation supported a given theoretical probability. The simulated probabilities, or estimated probabilities, suggested a workable process for understanding the probabilities. Only 50 trials were conducted in some examples; however, as stated in several other lessons, the more trials that are observed from a simulation, the better.

Problem Set

1. Some M&M's are "defective." For example, a defective M&M may have its "m" missing, or it may be cracked, broken, or oddly shaped. Is the probability of getting a defective M&M higher for peanut M&M's than for plain M&M's?

    Gloriann suggests the probability of getting a defective plain M&M is the same as the probability of getting a defective peanut M&M. Suzanne doesn't think this is correct because a peanut M&M is bigger than a plain M&M, and, therefore, has a greater opportunity to be damaged.

    a. Simulate inspecting a plain M&M by rolling two number cubes. Let a sum of 7 or 11 represent a defective plain M&M, and the other possible rolls represent a plain M&M that is not defective. Do 50 trials, and compute an estimate of the probability that a plain M&M is defective. Record the 50 outcomes you observed. Explain your process.

    b. Simulate inspecting a peanut M&M by selecting a card from a well-shuffled deck of cards. Let a one-eyed face card and clubs represent a defective peanut M&M, and the other cards represent a peanut M&M that is not defective. Be sure to replace the chosen card after each trial and to shuffle the deck well before choosing the next card. Note that the one-eyed face cards are the King of Diamonds, Jack of Hearts, and Jack of Spades. Do 20 trials, and compute an estimate of the probability that a peanut M&M is defective. Record the list of 20 cards that you observed. Explain your process.

    c. For this problem, suppose that the two simulations provide accurate estimates of the probability of a defective M&M for plain and peanut M&M's. Compare your two probability estimates, and decide whether Gloriann's belief is reasonable that the defective probability is the same for both types of M&M's. Explain your reasoning.

2. One at a time, mice are placed at the start of the maze shown below. There are four terminal stations at A, B, C, and D. At each point where a mouse has to decide in which direction to go, assume that it is equally likely for it to choose any of the possible directions. A mouse cannot go backwards.

    In the following simulated trials, L stands for Left, R for Right, and S for Straight. Estimate the probability that a mouse finds station C where the food is. No food is at A, B, or D. The following data were collected on 50 simulated paths that the mice took.
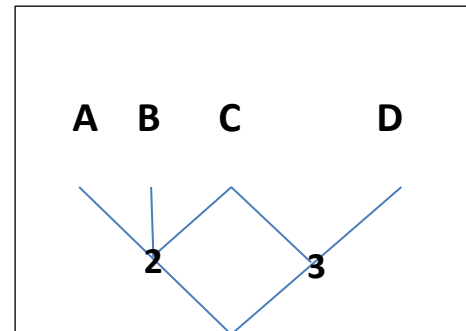
    LR RL RL LL LS LS RL RR RR RL

    RL LR LR RR LR LR LL LS RL LR

    RR LS RL RR RL LR LR LL LS RR

    RL RL RL RR RR RR LR LL LL RR

    RR LS RR LR RR RR LL RR LS LS

a. What paths constitute a success, and what paths constitute a failure?

b. Use the data to estimate the probability that a mouse finds food. Show your calculation.

c. Paige suggests that it is equally likely that a mouse gets to any of the four terminal stations. What does your simulation suggest about whether her equally likely model is believable? If it is not believable, what do your data suggest is a more believable model?

d. Does your simulation support the following theoretical probability model? Explain.

    i.   The probability a mouse finds terminal point A is 0.167.

    ii. The probability a mouse finds terminal point B is 0.167.

    iii. The probability a mouse finds terminal point C is 0.417.

    iv. The probability a mouse finds terminal point D is 0.250.