

Lesson 19: Understanding Variability When Estimating a Population Proportion

Classwork

In a previous lesson, you selected several random samples from a population. You recorded values of a numerical variable. You then calculated the mean for each sample, saw that there was variability in the sample means, and created a distribution of sample means to better see the sampling variability. You then considered larger samples and saw that the variability in the distribution decreased when the sample size increases. In this lesson, you will use a similar process to investigate variability in sample proportions.

Example 1: Sample Proportion

Your teacher will give your group a bag that contains colored cubes, some of which are red. With your classmates, you are going to build a distribution of sample proportions.

- a. Each person in your group should randomly select a sample of 10 cubes from the bag. Record the data for your sample in the table below.

Cube	Outcome (Color)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

- b. What is the proportion of red cubes in your sample of 10?

This value is called the sample proportion. The sample proportion is found by dividing the number of successes (in this example, the number of red cubes) by the total number of observations in the sample.

- c. Write your sample proportion on a post-it note, and place it on the number line that your teacher has drawn on the board. Place your note above the value on the number line that corresponds to your sample proportion.

The graph of all the students' sample proportions is called a sampling distribution of the sample proportions.

- d. Describe the shape of the distribution.

- e. Describe the variability in the sample proportions.

Based on the distribution, answer the following:

- f. What do you think is the population proportion?

- g. How confident are you of your estimate?

Example 2: Sampling Variability

What do you think would happen to the sampling distribution if everyone in class took a random sample of 30 cubes from the bag? To help answer this question, you will repeat the random sampling you did in Exercise 1, except now you will draw a random sample of 30 cubes instead of 10.

- a. Take a random sample of 30 cubes from the bag. Carefully record the outcome of each draw.

Cube	Outcome (Color)	Cube	Outcome (Color)
1		16	
2		17	
3		18	
4		19	
5		20	
6		21	
7		22	
8		23	
9		24	
10		25	
11		26	
12		27	
13		28	
14		29	
15		30	

- b. What is the proportion of red cubes in your sample of 30?
- c. Write your sample proportion on a post-it note, and place the note on the number line that your teacher has drawn on the board. Place your note above the value on the number line that corresponds to your sample proportion.
- d. Describe the shape of the distribution.

Exercises 1–5

1. Describe the variability in the sample proportions.
2. Based on the distribution, answer the following:
 - a. What do you think is the population proportion?
 - b. How confident are you of your estimate?
 - c. If you were taking a random sample of 30 cubes and determined the proportion that was red, do you think your sample proportion will be within 0.05 of the population proportion? Explain.
3. Compare the sampling distribution based on samples of size 10 to the sampling distribution based on samples of size 30.
4. As the sample size increased from 10 to 30, describe what happened to the sampling variability of the sample proportions.
5. What do you think would happen to the sampling variability of the sample proportions if the sample size for each sample was 50 instead of 30? Explain.

Lesson Summary

- The sampling distribution of the sample proportion is a graph of the sample proportions for many different samples.
- The mean of the sample proportions will be approximately equal to the value of the population proportion.
- As the sample size increases, the sampling variability decreases.

Problem Set

1. A class of seventh graders wanted to find the proportion of M&M's that are red. Each seventh grader took a random sample of 20 M&M's from a very large container of M&M's. Following is the proportion of red M&M's each student found.

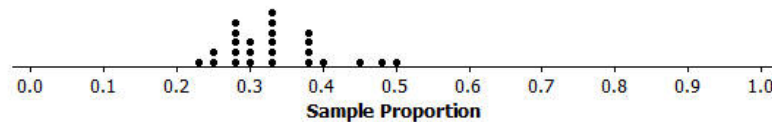
0.15	0	0.1	0.1	0.05	0.1	0.2	0.05	0.1
0.1	0.15	0.2	0	0.1	0.15	0.15	0.1	0.2
0.3	0.1	0.1	0.2	0.1	0.15	0.1	0.05	0.3

- a. Construct a dot plot of the sample proportions.
- b. Describe the shape of the distribution.
- c. Describe the variability of the distribution.
- d. Suppose the seventh-grade students had taken random samples of size 50. Describe how the sampling distribution would change from the one you constructed in part (a).

2. A group of seventh graders wanted to estimate the proportion of middle school students who suffer from allergies. The members of one group of seventh graders each took a random sample of 10 middle school students, and the members of another group of seventh graders each took a random sample of 40 middle school students. Below are two sampling distributions of the sample proportions of middle school students who said that they suffer from allergies. Which dot plot is based on random samples of size 40? How can you tell?

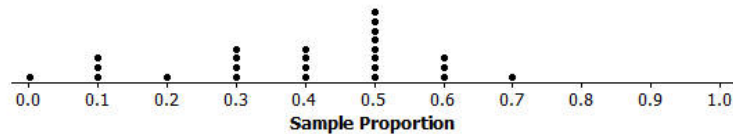
Dot Plot A:

Dot Plot of Sample Proportion



Dot Plot B:

Dot Plot of Sample Proportion



3. The nurse in your school district would like to study the proportion of middle school students who usually get at least eight hours of sleep on school nights. Suppose each student in your class plans on taking a random sample of 20 middle school students from your district, and each calculates a sample proportion of students who said that they usually get at least eight hours of sleep on school nights.
- Do you expect everyone in your class to get the same value for their sample proportion? Explain.
 - Suppose each student in class increased the sample size from 20 to 40. Describe how you could reduce the sampling variability.