Lesson 20: Estimating a Population Proportion

Classwork

In a previous lesson, each student in your class selected a random sample from a population and calculated the sample proportion. It was observed that there was sampling variability in the sample proportions, and as the sample size increased, the variability decreased. In this lesson, you will investigate how sample proportions can be used to estimate population proportions.

Example 1: Mean of Sample Proportions

A class of 30 seventh graders wanted to estimate the proportion of middle school students who were vegetarians. Each seventh grader took a random sample of 20 middle school students. Students were asked the question, "Are you a vegetarian?" One sample of 20 students had three students who said that they were vegetarians. For this sample, the sample proportion is $\frac{3}{20}$, or 0.15. Following are the proportions of vegetarians the seventh graders found in 30 samples. Each sample was of size 20 students. The proportions are rounded to the nearest hundredth.

0.15	0.10	0.15	0.00	0.10	0.15	0.10	0.10	0.05	0.20
0.25	0.15	0.25	0.25	0.30	0.20	0.10	0.20	0.05	0.10
0.10	0.30	0.15	0.05	0.25	0.15	0.20	0.10	0.20	0.15

Exercises 1–9

1. The first student reported a sample proportion of 0.15. Interpret this value in terms of the summary of the problem in the example.

2. Another student reported a sample proportion of 0. Did this student do something wrong when selecting the sample of middle school students?

3. Assume you were part of this seventh-grade class and you got a sample proportion of 0.20 from a random sample of middle school students. Based on this sample proportion, what is your estimate for the proportion of all middle school students who are vegetarians?

4. Construct a dot plot of the 30 sample proportions.

5. Describe the shape of the distribution.

6. Using the 30 class results listed above, what is your estimate for the proportion of all middle school students who are vegetarians? Explain how you made this estimate.

7. Calculate the mean of the 30 sample proportions. How close is this value to the estimate you made in Exercise 6?

8. The proportion of all middle school students who are vegetarians is 0.15. This is the actual proportion for the entire population of middle school students used to select the samples. How the mean of the 30 sample proportions compares with the actual population proportion depends on the students' samples.

9. Do the sample proportions in the dot plot tend to cluster around the value of the population proportion? Are any of the sample proportions far away from 0.15? List the proportions that are far away from 0.15.

Example 2: Estimating Population Proportion

Two hundred middle school students at Roosevelt Middle School responded to several survey questions. A printed copy of the responses the students gave to various questions will be provided by your teacher.

The data are organized in columns and are summarized by the following table:

Column Heading	Description
ID	Numbers from 1 to 200
Travel to School	Method used to get to school: Walk, car, rail, bus, bike, skateboard, boat, other
Favorite Season	Summer, fall, winter, spring
Allergies	Yes or no
Favorite School Subject	Art, English, languages, social studies, history, geography, music, science, computers, math, PE, other
Favorite Music	Classical, country, heavy metal, jazz, pop, punk rock, rap, reggae, R&B, rock and roll, techno, gospel, other
What superpower would you like?	Invisibility, super strength, telepathy, fly, freeze time

The last column in the data file is based on the question: Which of the following superpowers would you most like to have? The choices were invisibility, super strength, telepathy, the ability to fly, or the ability to freeze time.

The class wants to determine the proportion of Roosevelt Middle School students who answered freeze time to the last question. You will use a sample of the Roosevelt Middle School population to estimate the proportion of the students who answered freeze time to the last question.

A random sample of 20 student responses is needed. You are provided the random number table you used in a previous lesson. A printed list of the 200 Roosevelt Middle School students is also provided. In small groups, complete the following exercise:

a. Select a random sample of 20 student responses from the data file. Explain how you selected the random sample.

b. In the table below, list the 20 responses for your sample.

	Response
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

c. Estimate the population proportion of students who responded "freeze time" by calculating the sample proportion of the 20 sampled students who responded "freeze time" to the question.

d. Combine your sample proportion with other students' sample proportions, and create a dot plot of the distribution of the sample proportions of students who responded "freeze time" to the question.

e. By looking at the dot plot, what is the value of the proportion of the 200 Roosevelt Middle School students who responded "freeze time" to the question?

f. Usually you will estimate the proportion of Roosevelt Middle School students using just a single sample proportion. How different was your sample proportion from your estimate based on the dot plot of many samples?

g. Circle your sample proportion on the dot plot. How does your sample proportion compare with the mean of all the sample proportions?

h. Calculate the mean of all of the sample proportions. Locate the mean of the sample proportions in your dot plot; mark this position with an X. How does the mean of the sample proportions compare with your sample proportion?

Lesson Summary

The sample proportion from a random sample can be used to estimate a population proportion. The sample proportion will not be exactly equal to the population proportion, but values of the sample proportion from random samples tend to cluster around the actual value of the population proportion.

Problem Set

1. A class of 30 seventh graders wanted to estimate the proportion of middle school students who played a musical instrument. Each seventh grader took a random sample of 25 middle school students and asked each student whether or not they played a musical instrument. Following are the sample proportions the seventh graders found in 30 samples.

0.80	0.64	0.72	0.60	0.60	0.72	0.76	0.68	0.72	0.68
0.72	0.68	0.68	0.76	0.84	0.60	0.80	0.72	0.76	0.80
0.76	0.60	0.80	0.84	0.68	0.68	0.70	0.68	0.64	0.72

   a. The first student reported a sample proportion of 0.80. What does this value mean in terms of this scenario?

   b. Construct a dot plot of the 30 sample proportions.

   c. Describe the shape of the distribution.

   d. Describe the variability of the distribution.

   e. Using the 30 class sample proportions listed above, what is your estimate for the proportion of all middle school students who played a musical instrument?

2. Select another variable or column from the data file that is of interest. Take a random sample of 30 students from the list, and record the response to your variable of interest of each of the 30 students.

   a. Based on your random sample, what is your estimate for the proportion of all middle school students?

   b. If you selected a second random sample of 30, would you get the same sample proportion for the second random sample that you got for the first random sample? Explain why or why not.