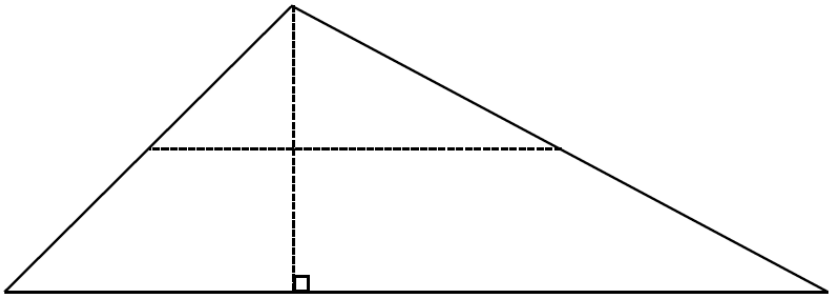
Lesson 25: Volume of Right Prisms

Classwork

Opening Exercise

Take your copy of the following figure, and cut it into four pieces along the dotted lines (the vertical line is the altitude, and the horizontal line joins the midpoints of the two sides of the triangle).

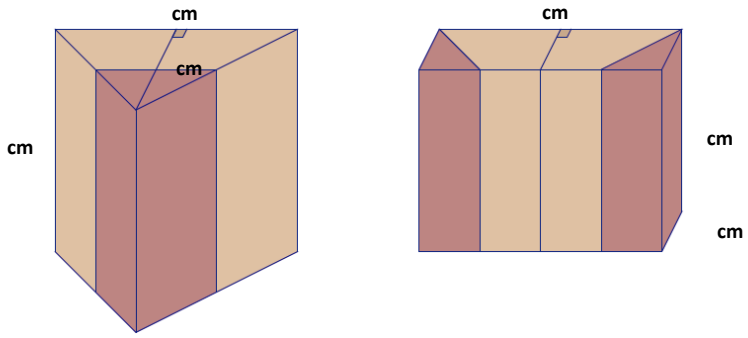
Arrange the four pieces so that they fit together to form a rectangle.



If a prism were formed out of each shape, the original triangle and your newly rearranged rectangle, and both prisms had the same height, would they have the same volume? Discuss with a partner.

Exercise 1

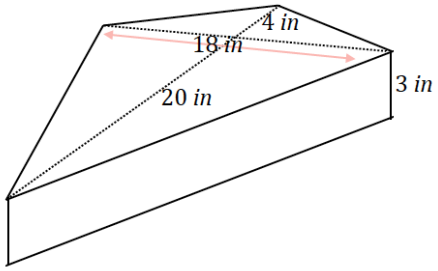
a. Show that the following figures have equal volumes.



b. How can it be shown that the prisms will have equal volumes without completing the entire calculation?

Example 1

Calculate the volume of the following prism.



Example 2

A container is shaped like a right pentagonal prism with an open top. When a cubic foot of water is dumped into the container, the depth of the water is inches. Find the area of the pentagonal base.

Example 3

Two containers are shaped like right triangular prisms each with the same height. The base area of the larger container is more than the base area of the smaller container. How many times must the smaller container be filled with water and poured into the larger container in order to fill the larger container?

Exercise 3

Two aquariums are shaped like right rectangular prisms. The ratio of the dimensions of the larger aquarium to the dimensions of the smaller aquarium is .

Addie says the larger aquarium holds more water than the smaller aquarium.

Berry says that the larger aquarium holds more water.

Cathy says that the larger aquarium holds over more water.

Are any of the girls correct? Explain your reasoning.

Problem Set

1. The pieces in Figure 1 are rearranged and put together to form Figure 2.

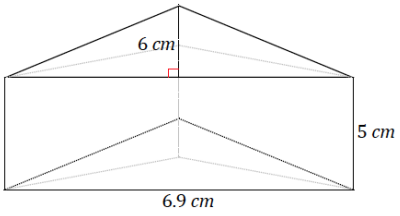


Figure 1

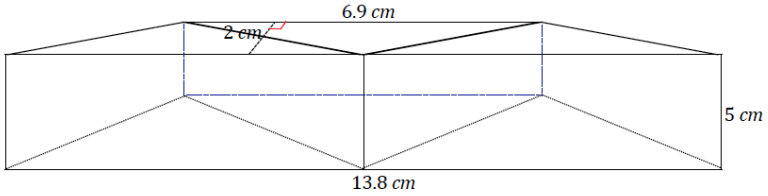


Figure 2

   a. Use the information in Figure 1 to determine the volume of the prism.
   b. Use the information in Figure 2 to determine the volume of the prism.
   c. If we were not told that the pieces of Figure 1 were rearranged to create Figure 2, would it be possible to determine whether the volumes of the prisms were equal without completing the entire calculation for each?

2. Each of two right prism containers is filled with gallons of water. The depth of the water in the first container is inches. The depth of the water in the second container is inches. If the area of the base in the first container is ft^2, find the area of the base in the second container. Explain your reasoning.

3. Two containers are shaped like right rectangular prisms. Each has the same height, but the base of the larger container is more in each direction. If the smaller container holds gallons when full, how many gallons does the larger container hold? Explain your reasoning.

4. A right prism container with the base area of ft^2 and height of ft. is filled with water until it is ft. deep. If a solid cube with edge length ft. is dropped to the bottom of the container, how much will the water rise?

5. A right prism container with a base area of ft^2 and height ft. is filled with water until it is ft. deep. A large boulder is dropped to the bottom of the container, and the water rises to the top completely submerging the boulder and without causing overflow. Find the volume of the boulder.

6. A rectangular swimming pool is feet wide and feet long. The rectangular floor of the swimming pool is feet wide, feet deep at one end, and feet deep at the other.
   a. Sketch the swimming pool as a right prism.
   b. What kind of right prism is the swimming pool?
   c. What is the volume of the swimming pool in cubic feet?
   d. How many gallons will the swimming pool hold if each cubic feet of water is about gallons?

7. A milliliter (mL) has volume of cm^3. A mL measuring cup is filled to mL. A small stone is placed in the measuring cup. The stone is completely submerged and the water level rises to mL.
   a. What is the volume of the stone in cm^3?
   b. Describe a right rectangular prism that has the same volume as the stone.