Lesson 3: Numbers in Exponential Form Raised to a Power

Classwork

For any number x and any positive integers m and n,

$$x^{m\ \ n} = x^{mn}$$

because

$$\begin{aligned} x^{m\ \ n} &= \underbrace{x \cdot x \cdots x}_{m \text{ times}}{}^{n} \\ &= \underbrace{\underbrace{x \cdot x \cdots x}_{m \text{ times}} \times \cdots \times \underbrace{x \cdot x \cdots x}_{m \text{ times}}}_{n \text{ times}} \\ &= x^{mn}. \end{aligned}$$

Exercise 1

$15^{3\ \ 9} =$

Exercise 2

$-2\ ^{5\ \ 8} =$

Exercise 3

$3.4^{17\ \ 4} =$

Exercise 4

Let s be a number.

$s^{17\ \ 4} =$

Exercise 5

Sarah wrote $3^{5\ \ 7} = 3^{12}$. Correct her mistake. Write an exponential expression using a base of 3 and exponents of 5, 7, and 12 that would make her answer correct.

Exercise 6

A number y satisfies $y^{24} - 256 = 0$. What equation does the number $x = y^4$ satisfy?

For any numbers x and y, and positive integer n,

$$(xy)^n = x^n y^n$$

because

$$\begin{aligned}(xy)^n &= \underbrace{(xy)\cdots(xy)}_{n \text{ times}} \\ &= \underbrace{x \cdot x \cdots x}_{n \text{ times}} \cdot \underbrace{y \cdot y \cdots y}_{n \text{ times}} \\ &= x^n y^n.\end{aligned}$$

Exercise 7

$(11 \times 4)^9 =$

Exercise 8

$(3^2 \times 7^4)^5 =$

Exercise 9

Let a, b, and c be numbers.

$(3^2 a^4)^5 =$

Exercise 10

Let x be a number.

$(5x)^7 =$

Exercise 11

Let x and y be numbers.

$(5xy^2)^7 =$

Exercise 12

Let a, b, and c be numbers.

$(a^2 b c^3)^4 =$

Exercise 13

Let x and y be numbers, $y \neq 0$, and let n be a positive integer. How is $\left(\frac{x}{y}\right)^n$ related to x^n and y^n?

Problem Set

1. Show (prove) in detail why $2 \cdot 3 \cdot 7^{\ 4} = 2^4 3^4 7^4$.

2. Show (prove) in detail why $xyz^{\ 4} = x^4 y^4 z^4$ for any numbers x, y, z.

3. Show (prove) in detail why $xyz^{\ n} = x^n y^n z^n$ for any numbers x, y, and z and for any positive integer n.