Lesson 19: The Graph of a Linear Equation in Two Variables Is a Line

Classwork

Exercises

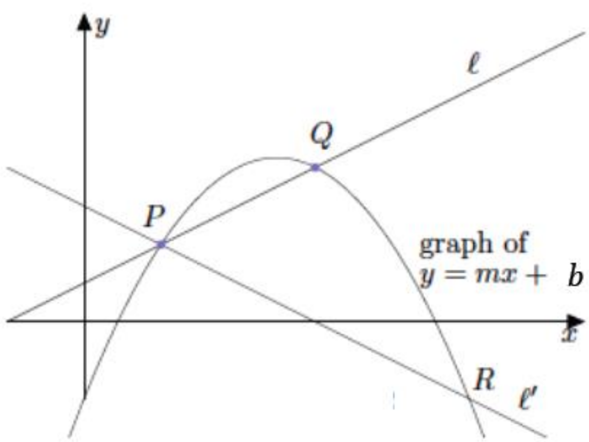
THEOREM: The graph of a linear equation $y = mx + b$ is a non-vertical line with slope m and passing through $(0, b)$, where b is a constant.

1. Prove the theorem by completing parts (a)–(c). Given two distinct points, P and Q, on the graph of $y = mx + b$ and let l be the line passing through P and Q. You must show the following:

   (1) Any point on the graph of $y = mx + b$ is on line l, and

   (2) Any point on the line l is on the graph of $y = mx + b$.

   a. Proof of (1): Let R be any point on the graph of $y = mx + k$. Show that R is on l. Begin by assuming it is not. Assume the graph looks like the diagram below where R is on l'.

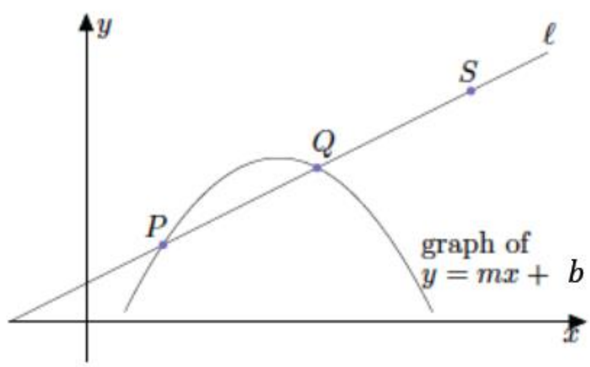


   What is the slope of line l?

What is the slope of line l'?

What can you conclude about lines l and l'? Explain.

b. Proof of (2): Let S be any point on line l, as shown.



Show that S is a solution to $y = mx + b$. Hint: Use the point $(0, b)$.

c. Now that you have shown that any point on the graph of $y = mx + b$ is on line l in part (a), and any point on line l is on the graph of $y = mx + b$ in part (b), what can you conclude about the graphs of linear equations?

2. Use $x = 4$ and $x = -4$ to find two solutions to the equation $x + 2y = 6$. Plot the solutions as points on the coordinate plane, and connect the points to make a line.

   a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $x + 2y = 6$.

   b. When $x = 1$, what is the value of y? Does this solution appear to be a point on the line?

   c. When $x = -3$, what is the value of y? Does this solution appear to be a point on the line?

   d. Is the point $(3, 2)$ on the line?

   e. Is the point $(3, 2)$ a solution to the linear equation $x + 2y = 6$?

3. Use $x = 4$ and $x = 1$ to find two solutions to the equation $3x - y = 9$. Plot the solutions as points on the coordinate plane, and connect the points to make a line.

   a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $3x - y = 9$.

   b. When $x = 4.5$, what is the value of y? Does this solution appear to be a point on the line?

   c. When $x = \frac{1}{2}$, what is the value of y? Does this solution appear to be a point on the line?

   d. Is the point $(2, 4)$ on the line?

   e. Is the point $(2, 4)$ a solution to the linear equation $3x - y = 9$?

4. Use $x = 3$ and $x = -3$ to find two solutions to the equation $2x + 3y = 12$. Plot the solutions as points on the coordinate plane, and connect the points to make a line.

   a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $2x + 3y = 12$.

   b. When $x = 2$, what is the value of y? Does this solution appear to be a point on the line?

c. When $x = -2$, what is the value of y? Does this solution appear to be a point on the line?

d. Is the point $(8, -3)$ on the line?

e. Is the point $(8, -3)$ a solution to the linear equation $2x + 3y = 12$?

5. Use $x = 4$ and $x = -4$ to find two solutions to the equation $x - 2y = 8$. Plot the solutions as points on the coordinate plane and connect the points to make a line.

   a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $x - 2y = 8$.

   b. When $x = 7$, what is the value of y? Does this solution appear to be a point on the line?

   c. When $x = -3$, what is the value of y? Does this solution appear to be a point on the line?

d. Is the point $(-2, -3)$ on the line?

e. Is the point $(-2, -3)$ a solution to the linear equation $x - 2y = 8$?

6. Based on your work in Exercises 2–5, what conclusions can you draw about the points on a line and solutions to a linear equation?

7. Based on your work in Exercises 2–5, will a point that is not a solution to a linear equation be a point on the graph of a linear equation? Explain.

8. Based on your work in Exercises 2–5, what conclusions can you draw about the graph of a linear equation?

9. Graph the equation $-3x + 8y = 24$ using intercepts.

10. Graph the equation $x - 6y = 15$ using intercepts.

11. Graph the equation $4x + 3y = 21$ using intercepts.

Lesson Summary

The graph of a linear equation is a line. A linear equation can be graphed using two-points: the x-intercept and the y-intercept.

Example:

Graph the equation: $2x + 3y = 9$.

Replace x with zero and solve for y to determine the y-intercept.

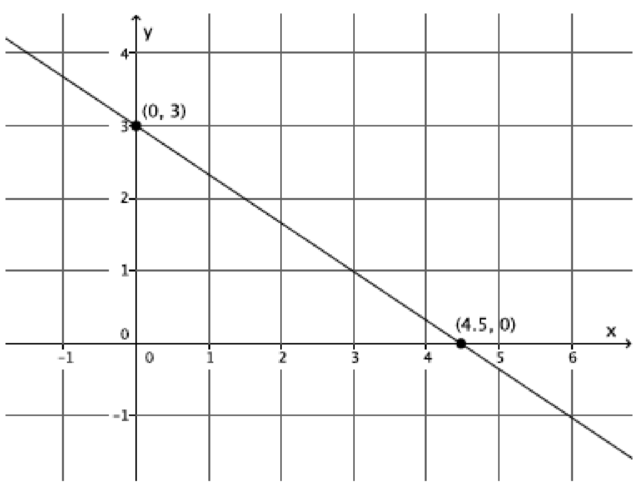
$$2(0) + 3y = 9$$
$$3y = 9$$
$$y = 3$$

The y-intercept is at $(0, 3)$.

Replace y with zero and solve for x to determine the x-intercept.

$$2x + 3(0) = 9$$
$$2x = 9$$
$$x = \frac{9}{2}$$

The x-intercept is at $\left(\frac{9}{2}, 0\right)$.



Problem Set

Graph each of the equations in the Problem Set on a different pair of x and y axes.

1. Graph the equation: $y = -6x + 12$.

2. Graph the equation: $9x + 3y = 18$.

3. Graph the equation: $y = 4x + 2$.

4. Graph the equation: $y = -\frac{5}{7}x + 4$.

5. Graph the equation: $\frac{3}{4}x + y = 8$.

6. Graph the equation: $2x - 4y = 12$.

7. Graph the equation: $y = 3$. What is the slope of the graph of this line?

8. Graph the equation: $x = -4$. What is the slope of the graph of this line?

9. Is the graph of $4x + 5y = \frac{3}{7}$ a line? Explain.

10. Is the graph of $6x^2 - 2y = 7$ a line? Explain.