# Lesson 1:  Modeling Linear Relationships

## Classwork

### Example 1:  Logging On

Lenore has just purchased a tablet computer, and she is considering purchasing an internet access plan so that she can connect to the Internet wirelessly from virtually anywhere in the world.  One company offers an internet access plan so that when a person connects to the company's wireless network, the person is charged a fixed access fee for connecting, PLUS an amount for the number of minutes connected based upon a constant usage rate in dollars per minute.

Lenore is considering this company's plan, but the company's advertisement does not state how much the fixed access fee for connecting is, nor does it state the usage rate.  However, the company's website says that a 10-minute session costs $0.40, a 20-minute session costs $0.70, and a 30-minute session costs $1.00.  Lenore decides that she will use these pieces of information to determine both the fixed access fee for connecting and the usage rate.

### Exercises 1–6

1.  Lenore makes a table of this information and a graph where *number of minutes* is represented by the horizontal axis and *total session cost* is represented by the vertical axis.  Plot the three given points on the graph.  These three points appear to lie on a line.  What information about the access plan suggests that the correct model is indeed a linear relationship?

| Number of Minutes | Total Session Cost |
|:---:|:---:|
| 0 | |
| 10 | $0.40 |
| 20 | $0.70 |
| 30 | $1.00 |
| 40 | |
| 50 | |
| 60 | |
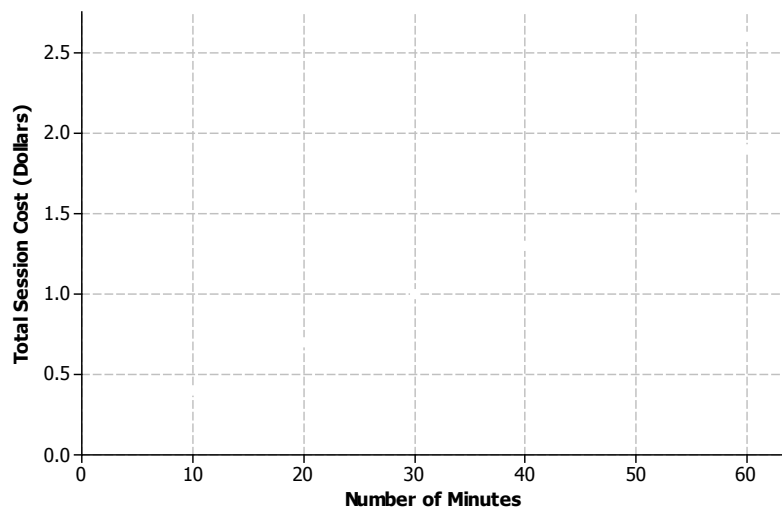
2.  The rate of change describes how the total cost changes with respect to time.

    a.  When the number of minutes increases by 10 (such as from 10 minutes to 20 minutes or from 20 minutes to 30 minutes), how much does the charge increase?

    b.  Another way to say this would be the "usage charge per 10 minutes of use." Use that information to determine the increase in cost based on only 1 minute of additional usage. In other words, find the "usage charge per minute of use."

3.  The company's pricing plan states that the usage rate is constant for any number of minutes connected to the Internet. In other words, the increase in cost for 10 more minutes of use (the value that you calculated above) will be the same whether you increase from 20 to 30 minutes, 30 to 40 minutes, etc. Using this information, determine the total cost for 40 minutes, 50 minutes, and 60 minutes of use. Record those values in the table, and plot the corresponding points on the graph in Exercise 1.

4.  Using the table and the graph in Exercise 1, compute the hypothetical cost for 0 minutes of use. What does that value represent in the context of the values that Lenore is trying to figure out?

5.  On the graph in Exercise 1, draw a line through the points representing 0 to 60 minutes of use under this company's plan. The slope of this line is equal to the rate of change, which in this case is the usage rate.

6.  Using $x$ for the number of minutes and $y$ for total cost in dollars, write a function to model the linear relationship between minutes of use and total cost.

## Example 2: Another Rate Plan

A second wireless access company has a similar method for computing its costs.  Unlike the first company that Lenore was considering, this second company explicitly states its access fee is $0.15, and its usage rate is $0.04 per minute.

$$\text{Total Session Cost} = \$0.15 + \$0.04(\text{number of minutes})$$

### Exercises 7–16

7.  Let $x$ represent the number of minutes used and $y$ represent the total session cost.  Construct a linear function that models the total session cost based on the number of minutes used.

8.  Using the linear function constructed in Exercise 7, determine the total session cost for sessions of 0, 10, 20, 30, 40, 50, and 60 minutes, and fill in these values in the table below.

| Number of Minutes | Total Session Cost |
|---|---|
| 0 | |
| 10 | |
| 20 | |
| 30 | |
| 40 | |
| 50 | |
| 60 | |

9.  Plot these points on the original graph in Exercise 1, and draw a line through these points.  In what ways does the line that represents this second company's access plan differ from the line that represented the first company's access plan?

MP3 download sites are a popular forum for selling music.  Different sites offer pricing that depend on whether or not you want to purchase an entire album or individual songs "à la carte."  One site offers MP3 downloads of individual songs with the following price structure:  a $3 fixed fee for monthly subscription PLUS a charge of $0.25 per song.

10. Using $x$ for the number of songs downloaded and $y$ for the total monthly cost, construct a linear function to model the relationship between the number of songs downloaded and the total monthly cost.

11. Construct a table to record the total monthly cost (in dollars) for MP3 downloads of 10 songs, 20 songs, and so on up to 100 songs.

|  |  |
|---|---|
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12. Plot the 10 data points in the table on a coordinate plane. Let the $x$-axis represent the number of songs downloaded and the $y$-axis represent the total monthly cost (in dollars) for MP3 downloads.

A band will be paid a flat fee for playing a concert.  Additionally, the band will receive a fixed amount for every ticket sold.  If 40 tickets are sold, the band will be paid $200.  If 70 tickets are sold, the band will be paid $260.

13.  Determine the rate of change.

14.  Let $x$ represent the number of tickets sold and $y$ represent the amount the band will be paid.  Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

15.  What is the fee the band will be paid for playing the concert (not including ticket sales)?

16.  How much will the band receive for each ticket sold?

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**Lesson Summary**

A linear function can be used to model a linear relationship between two types of quantities. The graph of a linear function is a straight line.

A linear function can be constructed using a rate of change and initial value. It can be interpreted as an equation of a line in which:

- The rate of change is the slope of the line and describes how one quantity changes with respect to another quantity.
- The initial value is the $y$-intercept.

## Problem Set

1. Recall that Lenore was investigating two wireless access plans. Her friend in Europe says that he uses a plan in which he pays a monthly fee of 30 euros plus 0.02 euros per minute of use.

   a. Construct a table of values for his plan's monthly cost based on 100 minutes of use for the month, 200 minutes of use, and so on up to 1,000 minutes of use. (The charge of 0.02 euros per minute of use is equivalent to 2 euros per 100 minutes of use.)

   b. Plot these 10 points on a carefully labeled graph, and draw the line that contains these points.

   c. Let $x$ represent minutes of use and $y$ represent the total monthly cost in euros. Construct a linear function that determines monthly cost based on minutes of use.

   d. Use the function to calculate the cost under this plan for 750 minutes of use. If you were to add this point to the graph, would it be above the line, below the line, or on the line?

2. A shipping company charges a \$4.45 handling fee in addition to \$0.27 per pound to ship a package.

   a. Using $x$ for weight in pounds and $y$ for the cost of shipping in dollars, write a linear function that determines the cost of shipping based on weight.

   b. Which line (solid, dotted, or dashed) on the graph below represents the shipping company's pricing method? Explain.
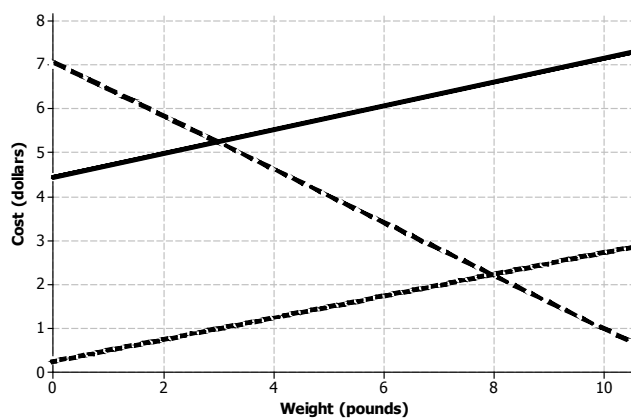
3. Kelly wants to add new music to her MP3 player. Another subscription site offers its downloading service using the following: Total Monthly Cost = 5.25 + 0.30(number of songs).

   a. Write a sentence (all words, no math symbols) that the company could use on its website to explain how it determines the price for MP3 downloads for the month.

   b. Let $x$ represent the number of songs downloaded and $y$ represent the total monthly cost in dollars. Construct a function to model the relationship between the number of songs downloaded and the total monthly cost.

   c. Determine the cost of downloading 10 songs.

4. Li Na is saving money. Her parents gave her an amount to start, and since then she has been putting aside a fixed amount each week. After six weeks, Li Na has a total of $82 made of her own savings in addition to the amount her parents gave her. Fourteen weeks from the start of the process, Li Na has $118.

   a. Using $x$ for the number of weeks and $y$ for the amount in savings (in dollars), construct a linear function that describes the relationship between the number of weeks and the amount in savings.

   b. How much did Li Na's parents give her to start?

   c. How much does Li Na set aside each week?

   d. Draw the graph of the linear function below (start by plotting the points for $x = 0$ and $x = 20$).
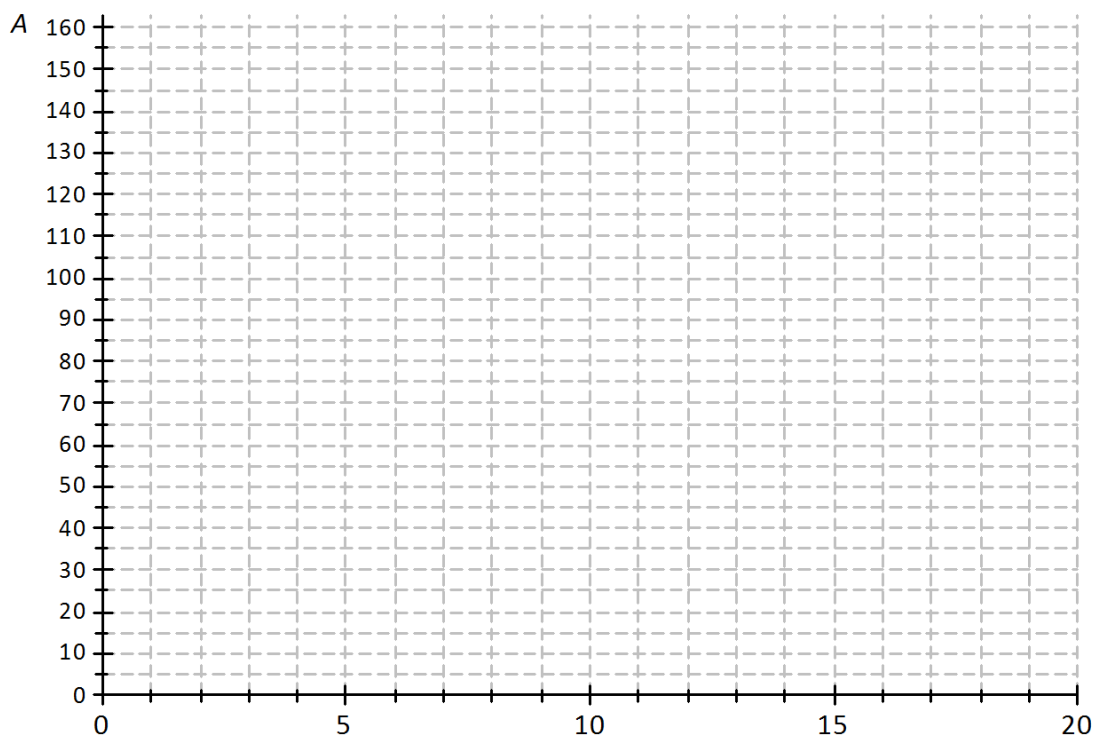
# Lesson 2:  Interpreting Rate of Change and Initial Value

## Classwork

Linear functions are defined by the equation of a line.  The graphs and the equations of the lines are important for understanding the relationship between the two variables represented in the following example as $x$ and $y$.

### Example 1:  Rate of Change and Initial Value

The equation of a line can be interpreted as defining a linear function.  The graphs and the equations of lines are important in understanding the relationship between two types of quantities (represented in the following examples by $x$ and $y$).

In a previous lesson, you encountered an MP3 download site that offers downloads of individual songs with the following price structure:  a \$3 fixed fee for monthly subscription PLUS a fee of \$0.25 per song.  The linear function that models the relationship between the number of songs downloaded and the total monthly cost of downloading songs can be written as

$$y = 0.25x + 3,$$

where $x$ represents the number of songs downloaded, and $y$ represents the total monthly cost (in dollars) for MP3 downloads.

    a.    In your own words, explain the meaning of 0.25 within the context of the problem.

    b.    In your own words, explain the meaning of 3 within the context of the problem.

The values represented in the function can be interpreted in the following way:

$$y = \underbrace{0.25x}_{\substack{\text{rate of} \\ \text{change}}} + \underbrace{3}_{\text{initial value}}$$

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| The coefficient of $x$ is referred to as the rate of change. It can be interpreted as the change in the values of $y$ for every one-unit increase in the values of $x$. When the rate of change is positive, the linear function is *increasing.* **In other words, *increasing* indicates that as the $x$-value increases, so does the $y$-value.** When the rate of change is negative, the linear function is *decreasing.* ***Decreasing* indicates that as the $x$-value increases, the $y$-value decreases.** | The constant value is referred to as the *initial value* or *y-intercept* and can be interpreted as the value of $y$ when $x = 0$. |
|---|---|

**Exercises 1–6:  Is It a Better Deal?**

Another site offers MP3 downloads with a different price structure:  a $2 fixed fee for monthly subscription PLUS a fee of $0.40 per song.

1.  Write a linear function to model the relationship between the number of songs downloaded and the total monthly cost.  As before, let $x$ represent the number of songs downloaded and $y$ represent the total monthly cost (in dollars) of downloading songs.

2.  Determine the cost of downloading 0 songs and 10 songs from this site.

3.  The graph below already shows the linear model for the first subscription site (Company 1):  $y = 0.25x + 3$.  Graph the equation of the line for the second subscription site (Company 2) by marking the two points from your work above (for 0 songs and 10 songs) and drawing a line through those two points.
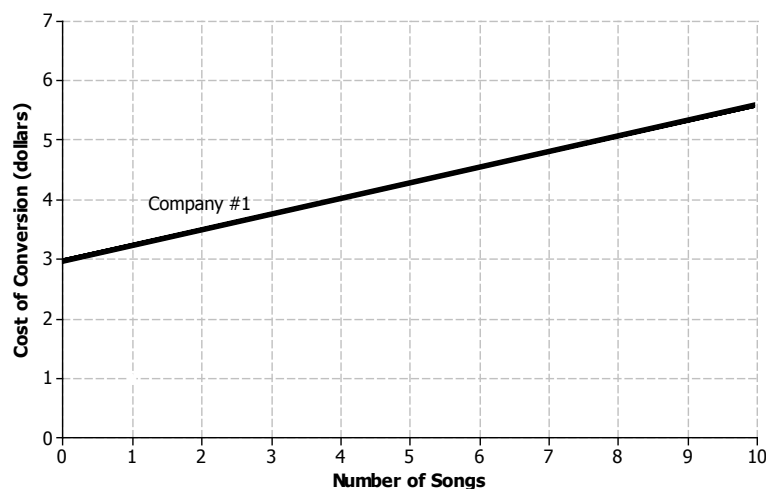
4.   Which line has a steeper slope?  Which company's model has the more expensive cost per song?

5.   Which function has the greater initial value?

6.   Which subscription site would you choose if you only wanted to download 5 songs per month?  Which company would you choose if you wanted to download 10 songs?  Explain your reasoning.

**Exercises 7–9:  Aging Autos**

7.   When someone purchases a new car and begins to drive it, the mileage (meaning the number of miles the car has traveled) immediately increases.  Let $x$ represent the number of years since the car was purchased and $y$ represent the total miles traveled.  The linear function that models the relationship between the number of years since purchase and the total miles traveled is $y = 15000x$.

a.   Identify and interpret the rate of change.

b.   Identify and interpret the initial value.

c.  Is the mileage increasing or decreasing each year according to the model?  Explain your reasoning.

8.  When someone purchases a new car and begins to drive it, generally speaking, the resale value of the car (in dollars) goes down each year.  Let $x$ represent the number of years since purchase and $y$ represent the resale value of the car (in dollars).  The linear function that models the resale value based on the number of years since purchase is $y = 20000 - 1200x$.

a.  Identify and interpret the rate of change.

b.  Identify and interpret the initial value.

c.  Is the resale value increasing or decreasing each year according to the model?  Explain.

9.  Suppose you are given the linear function $y = 2.5x + 10$.

a.  Write a story that can be modeled by the given linear function.

b.  What is the rate of change?  Explain its meaning with respect to your story.

c.  What is the initial value?  Explain its meaning with respect to your story.

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**Lesson Summary**

When a linear function is given by the equation of a line of the form $y = mx + b$, the rate of change is $m$ and initial value is $b$. Both are easy to identify.

The rate of change of a linear function is the slope of the line it represents. It is the change in the values of $y$ per a one-unit increase in the values of $x$.

- A positive rate of change indicates that a linear function is increasing.
- A negative rate of change indicates that a linear function is decreasing.
- Given two lines each with positive slope, the function represented by the steeper line has a greater rate of change.

The initial value of a linear function is the value of the $y$-variable when the $x$-value is zero.

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## Problem Set

1. A rental car company offers the following two pricing methods for its customers to choose from for a one-month rental:

    Method 1: Pay $400 for the month, or

    Method 2: Pay $0.30 per mile plus a standard maintenance fee of $35.

    a. Construct a linear function that models the relationship between the miles driven and the total rental cost for Method 2. Let $x$ represent the number of miles driven and $y$ represent the rental cost (in dollars).

    b. If you plan to drive 1,100 miles for the month, which method would you choose? Explain your reasoning.

2. Recall from a previous lesson that Kelly wants to add new music to her MP3 player. She was interested in a monthly subscription site that offered its MP3 downloading service for a monthly subscription fee PLUS a fee per song. The linear function that modeled the total monthly cost ($y$) based on the number of songs downloaded ($x$) is $y = 5.25 + 0.30x$.

    The site has suddenly changed its monthly price structure. The linear function that models the new total monthly cost ($y$) based on the number of songs downloaded ($x$) is $y = 0.35x + 4.50$.

    a. Explain the meaning of the new 4.50 value in the equation. Is this a better situation for Kelly than before?

    b. Explain the meaning of the new 0.35 value in the equation. Is this a better situation for Kelly than before?

    c. If you were to graph the two equations (old vs. new), which line would have the steeper slope? What does this mean in the context of the problem?

    d. Which subscription plan provides the best value if Kelly will download fewer than 15 songs per month?

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Lesson 2: Interpreting Rate of Change and Initial Value
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# Lesson 3: Representations of a Line

## Classwork

### Example 1: Rate of Change and Initial Value Given in the Context of the Problem

A truck rental company charges a $150 rental fee in addition to a charge of $0.50 per mile driven. In this problem, you will graph the linear function relating the total cost of the rental in dollars, $C$, to the number of miles driven, $m$, on the axes below.
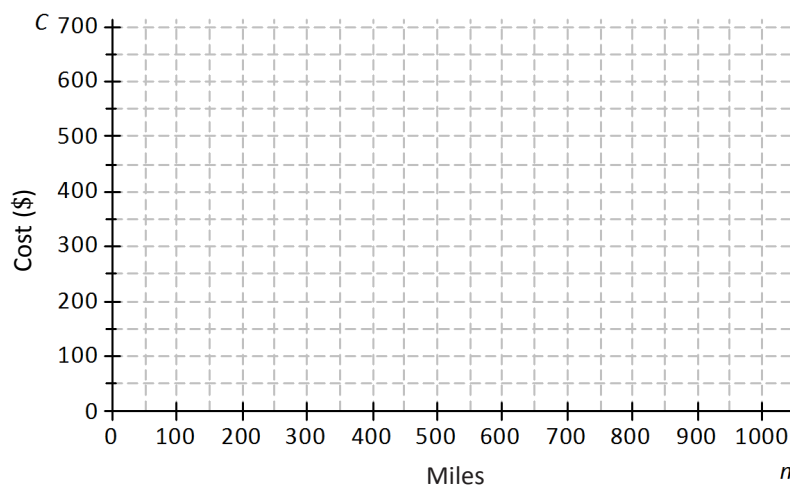


a. If the truck is driven 0 miles, what will be the cost to the customer? How will this be shown on the graph?

b. What is the rate of change that relates cost to number of miles driven? Explain what it means within the context of the problem.

c. On the axes given, sketch the graph of the linear function that relates $C$ to $m$.

d. Write the equation of the linear function that models the relationship between number of miles driven and total rental cost.

## Exercises

Jenna bought a used car for $18,000.  She has been told that the value of the car is likely to decrease by $2,500 for each year that she owns the car.  Let the value of the car in dollars be $V$ and the number of years Jenna has owned the car be $t$.
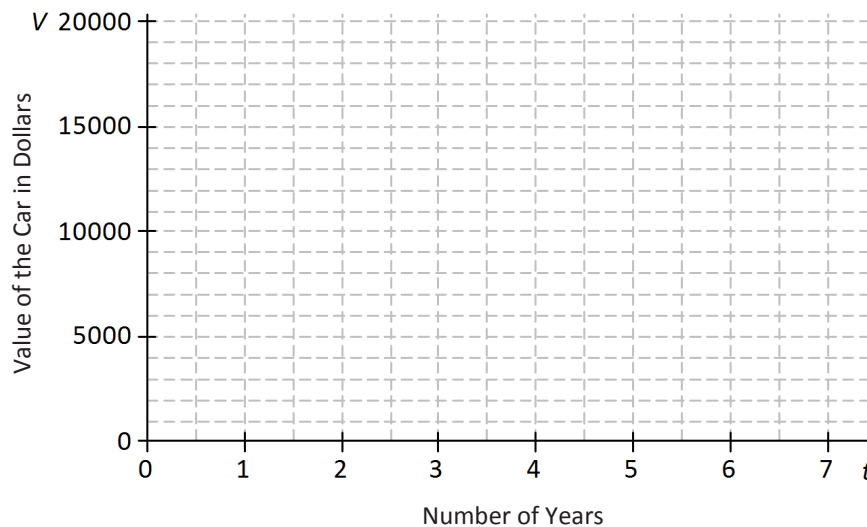


1.  What is the value of the car when $t = 0$?  Show this point on the graph.


2.  What is the rate of change that relates $V$ to $t$?  (Hint:  Is it positive or negative?  How can you tell?)


3.  Find the value of the car when
    a.  $t = 1$.

    b.  $t = 2$.

    c.  $t = 7$.

4.  Plot the points for the values you found in Exercise 3, and draw the line (using a straight-edge) that passes through those points.

5.  Write the linear function that models the relationship between the number of years Jenna has owned the car and the value of the car.

An online bookseller has a new book in print. The company estimates that if the book is priced at $15, then 800 copies of the book will be sold per day, and if the book is priced at $20, then 550 copies of the book will be sold per day.
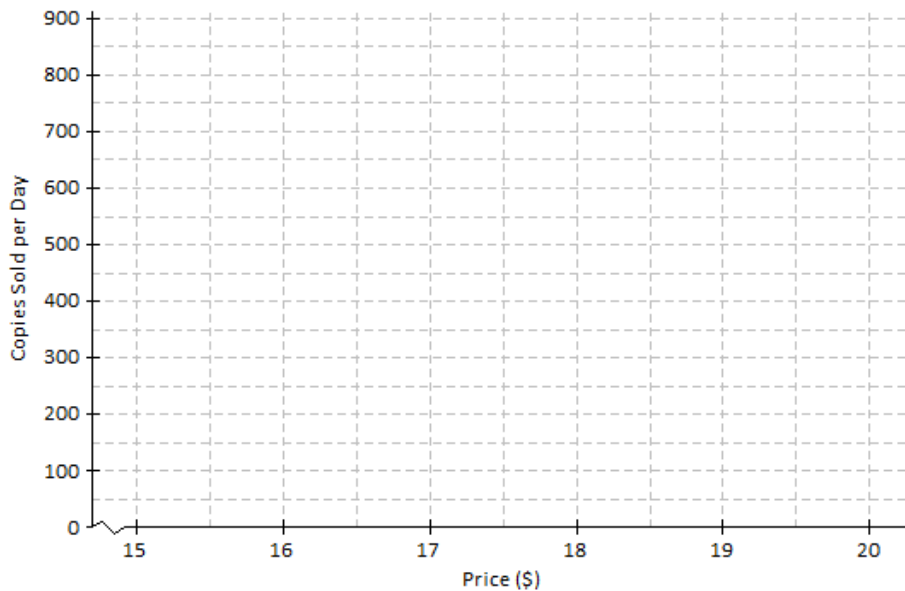


6. Identify the ordered pairs given in the problem. Then, plot both on the graph.

7. Assume that the relationship between the number of books sold and the price is linear. (In other words, assume that the graph is a straight line.) Using a straight-edge, draw the line that passes through the two points.

8. What is the rate of change relating number of copies sold to price?

9. Based on the graph, if the company prices the book at $18, about how many copies of the book can they expect to sell per day?

10. Based on the graph, approximately what price should the company charge in order to sell 700 copies of the book per day?

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**Lesson Summary**

When the rate of change, $b$, and an initial value, $a$, are given in the context of a problem, the linear function that models the situation is given by the equation $y = a + bx$.

The rate of change and initial value can also be used to sketch the graph of the linear function that models the situation.

When two or more ordered pairs are given in the context of a problem that involves a linear relationship, the graph of the linear function is the line that passes through those points. The linear function can be represented by the equation of that line.

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## Problem Set

1.  A plumbing company charges a service fee of $120, plus $40 for each hour worked. In this problem, you will sketch the graph of the linear function relating the cost to the customer (in dollars), $C$, to the time worked by the plumber (in hours), $t$, on the axes below.
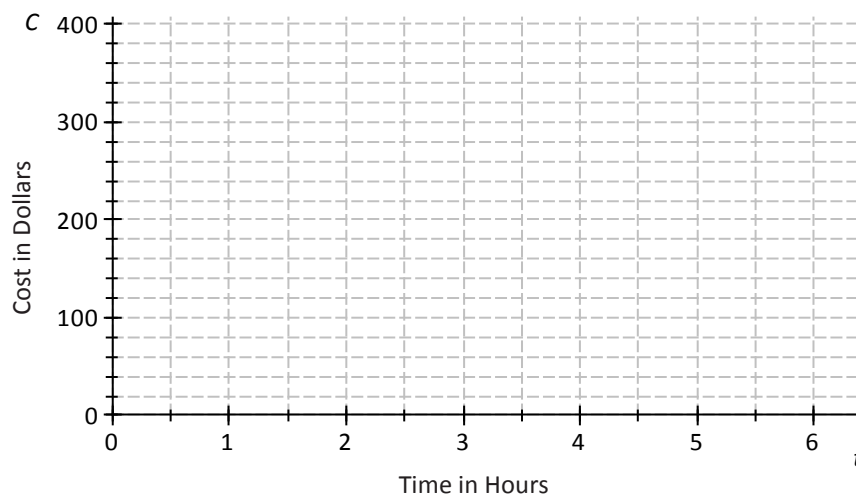


a.  If the plumber works for 0 hours, what will be the cost to the customer? How will this be shown on the graph?

b.  What is the rate of change that relates cost to time?

c.  Write a linear function that models the relationship between the hours worked and cost to the customer.

d.  Find the cost to the customer if the plumber works for each of the following number of hours.

    i.   1 hour

    ii.  2 hours

    iii. 6 hours

e.  Plot the points for these times on the coordinate plane, and use a straight-edge to draw the line through the points.

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2. An author has been paid a writer's fee of $1,000 and will additionally receive $1.50 for every copy of the book that is sold.

   a. Sketch the graph of the linear function that relates the total amount of money earned, $A$, to the number of books sold, $n$, on the axes below.
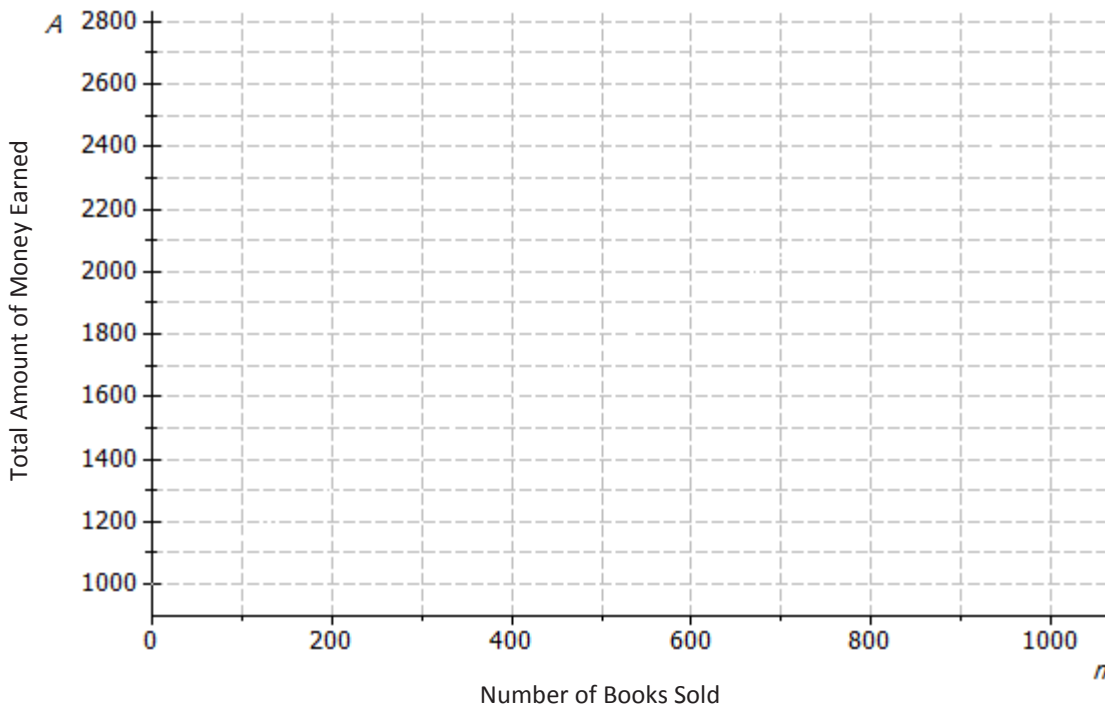


   b. What is the rate of change that relates the total amount of money earned to the number of books sold?

   c. What is the initial value of the linear function based on the graph?

   d. Let the number of books sold be $n$ and the total amount earned be $A$. Construct a linear function that models the relationship between the number of books sold and the total amount earned.

3. Suppose that the price of gasoline has been falling. At the beginning of last month ($t = 0$), the price was \$4.60 per gallon. Twenty days later ($t = 20$), the price was \$4.20 per gallon. Assume that the price per gallon, $P$, fell at a constant rate over the twenty days.
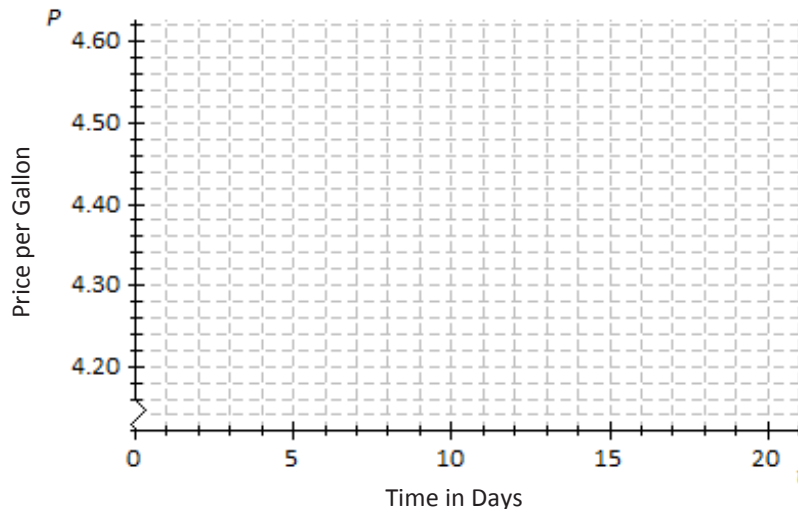


a. Identify the ordered pairs given in the problem. Plot both points on the coordinate plane above.

b. Using a straight-edge, draw the line that contains the two points.

c. What is the rate of change? What does it mean within the context of the problem?

d. What is the function that models the relationship between the number of days and the price per gallon?

e. What was the price of gasoline after 9 days?

f. After how many days was the price \$4.32?

# Lesson 4: Increasing and Decreasing Functions

## Classwork

Graphs are useful tools in terms of representing data. They provide a visual story, highlighting important facts that surround the relationship between quantities.
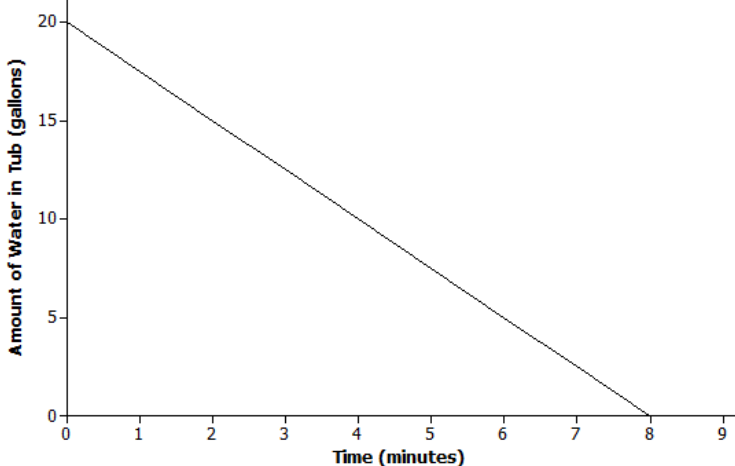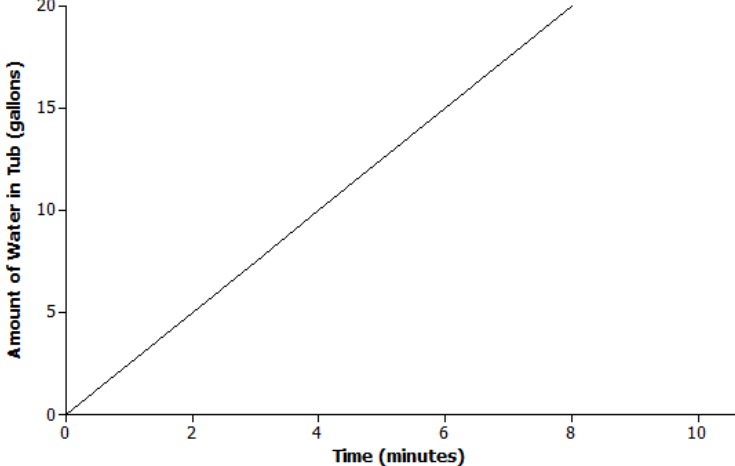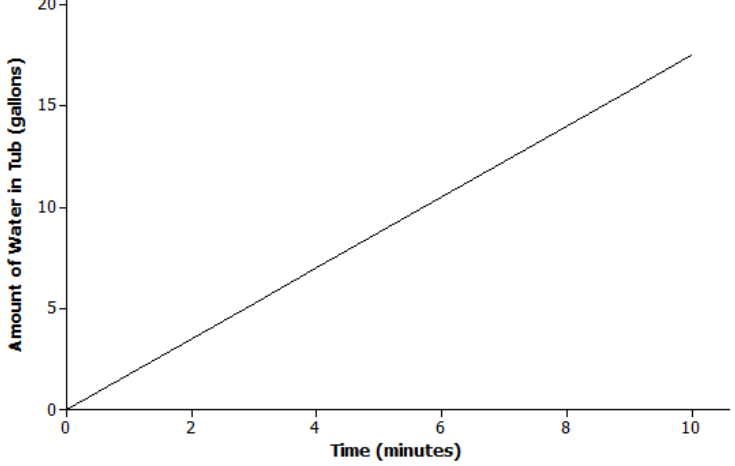
The graph of a linear function is a line. The slope of the line can provide useful information about the functional relationship between the two types of quantities:

- A linear function whose graph has a positive slope is said to be an *increasing function.*
- A linear function whose graph has a negative slope is said to be a *decreasing function.*
- A linear function whose graph has a zero slope is said to be a *constant function.*

**Exercises**

1. Read through each of the scenarios and choose the graph of the function that best matches the situation. Explain the reason behind each choice.

   a. A bathtub is filled at a constant rate of 1.75 gallons per minute.

   b. A bathtub is drained at a constant rate of 2.5 gallons per minute.

   c. A bathtub contains 2.5 gallons of water.

   d. A bathtub is filled at a constant rate of 2.5 gallons per minute.



**Scenario:**
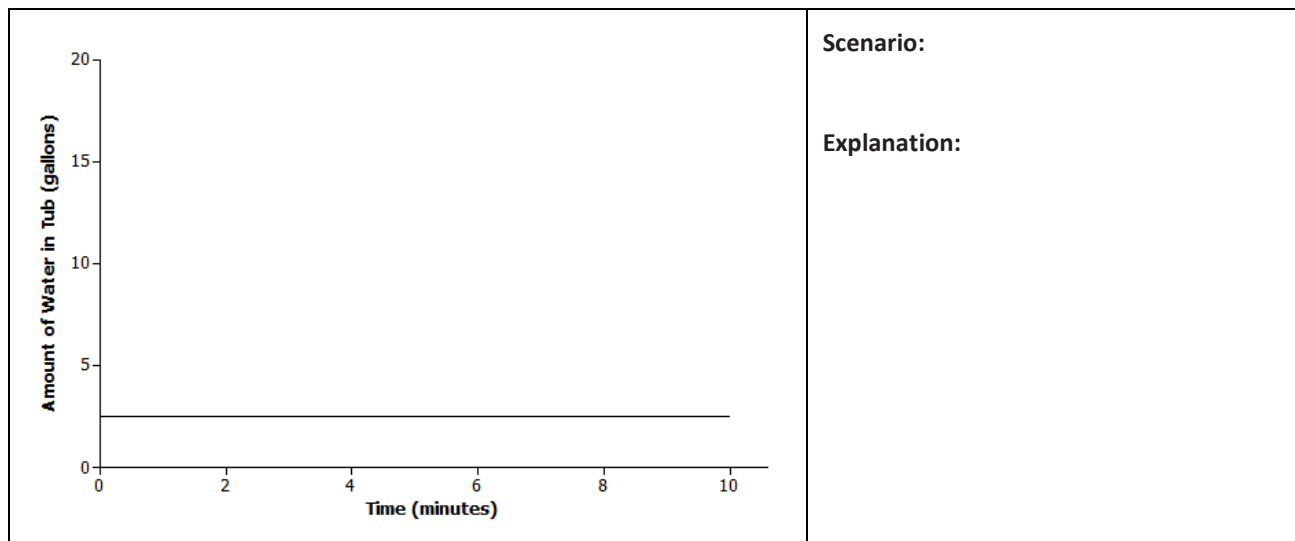
**Explanation:**

**Scenario:**

**Explanation:**



**Scenario:**

**Explanation:**



**Scenario:**

**Explanation:**

2.  Read through each of the scenarios, and sketch a graph of a function that models the situation.

    a.  A messenger service charges a flat rate of $4.95 to deliver a package regardless of distance to the destination.
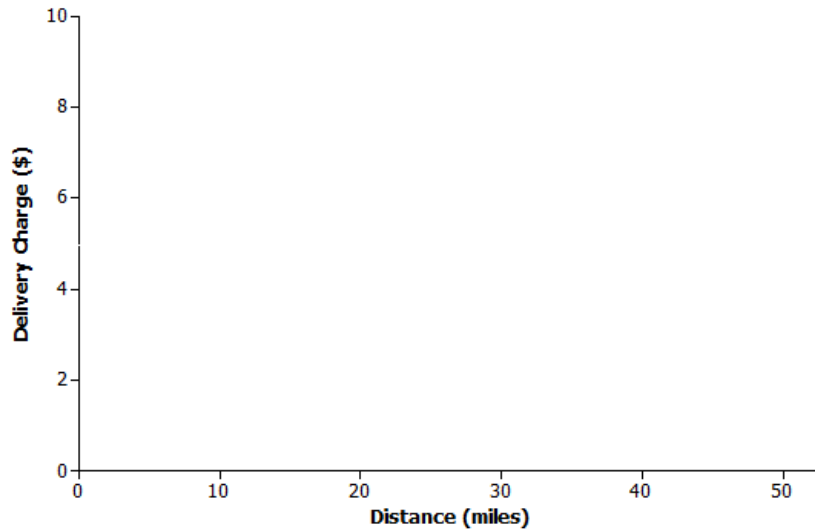


    b.  At sea level, the air that surrounds us presses down on our bodies at 14.7 pounds per square inch (psi).  For every 10 meters that you dive under water, the pressure increases by 14.7 psi.
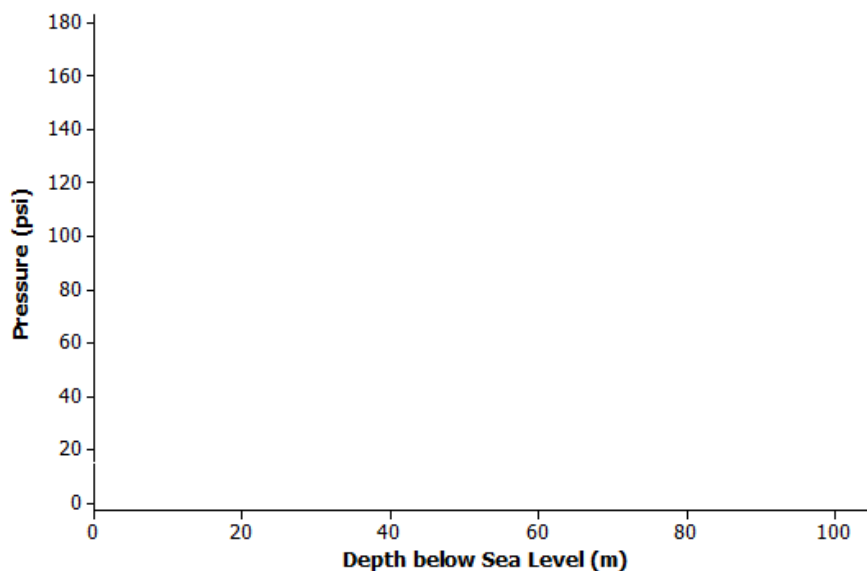
c.  The range (driving distance per charge) of an electric car varies based on the average speed the car is driven. The initial range of the electric car after a full charge is 400 miles.  However, the range is reduced by 20 miles for every 10 mph increase in average speed the car is driven.
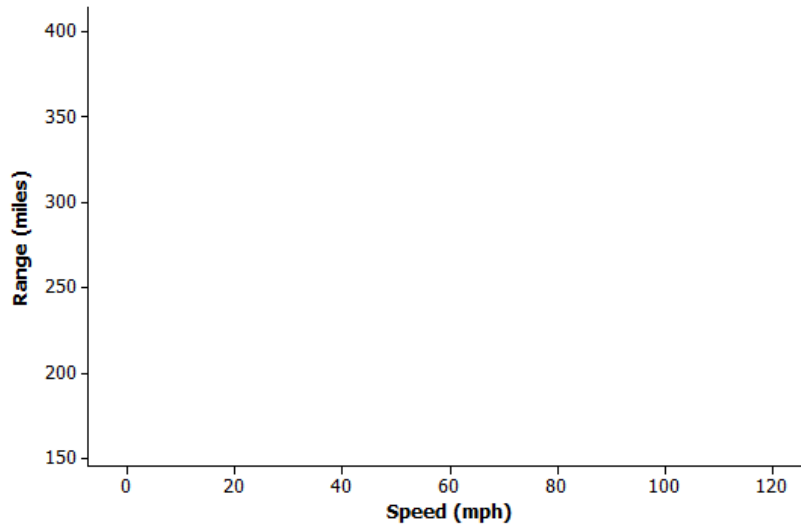


3.  The graph below represents the total number of smart phones that are shipped to a retail store over the course of 50 days.
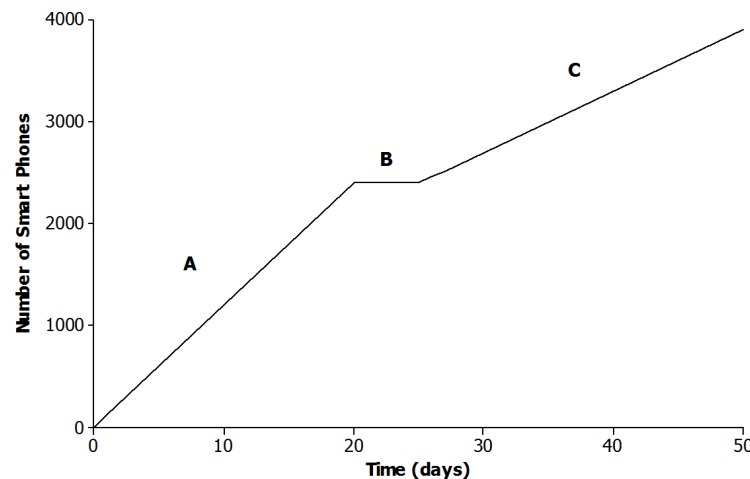


Match each part of the graph (A, B, and C) to its verbal description.  Explain the reasoning behind your choice.

  i.  Half of the factory workers went on strike, and not enough smartphones were produced for normal shipments.

    ii.    The production schedule was normal, and smartphones were shipped to the retail store at a constant rate.

    iii.    A defective electronic chip was found, and the factory had to shut down; so, no smartphones were shipped.

4.   The relationship between Jameson's account balance and time is modeled by the graph below.
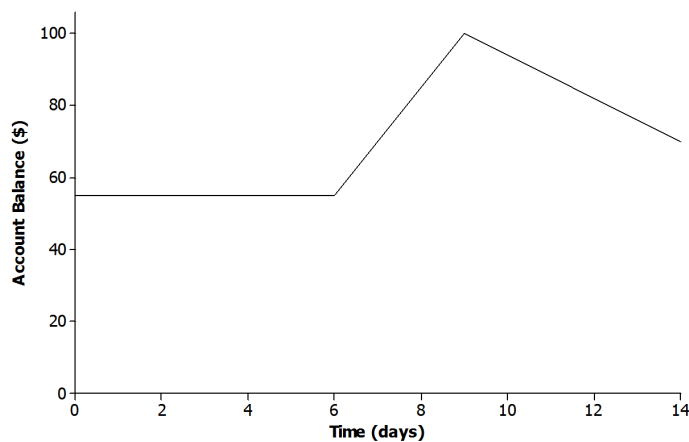


   a.   Write a story that models the situation represented by the graph.

   b.   When is the function represented by the graph increasing?  How does this relate to your story?

   c.   When is the function represented by the graph decreasing?  How does this relate to your story?

> **Lesson Summary**
>
> The graph of a function can be used to help describe the relationship between two quantities.
>
> The slope of the line can provide useful information about the functional relationship between two quantities:
>
> - A function whose graph has a positive slope is said to be an *increasing function.*
> - A function whose graph has a negative slope is said to be a *decreasing function.*
> - A function whose graph has a zero slope is said to be a *constant function.*

## Problem Set

1. Read through each of the scenarios, and choose the graph of the function that best matches the situation. Explain the reason behind each choice.

   a. The tire pressure on Regina's car remains at 30 psi.

   b. Carlita inflates her tire at a constant rate for 4 minutes.

   c. Air is leaking from Courtney's tire at a constant rate.



**Scenario:**
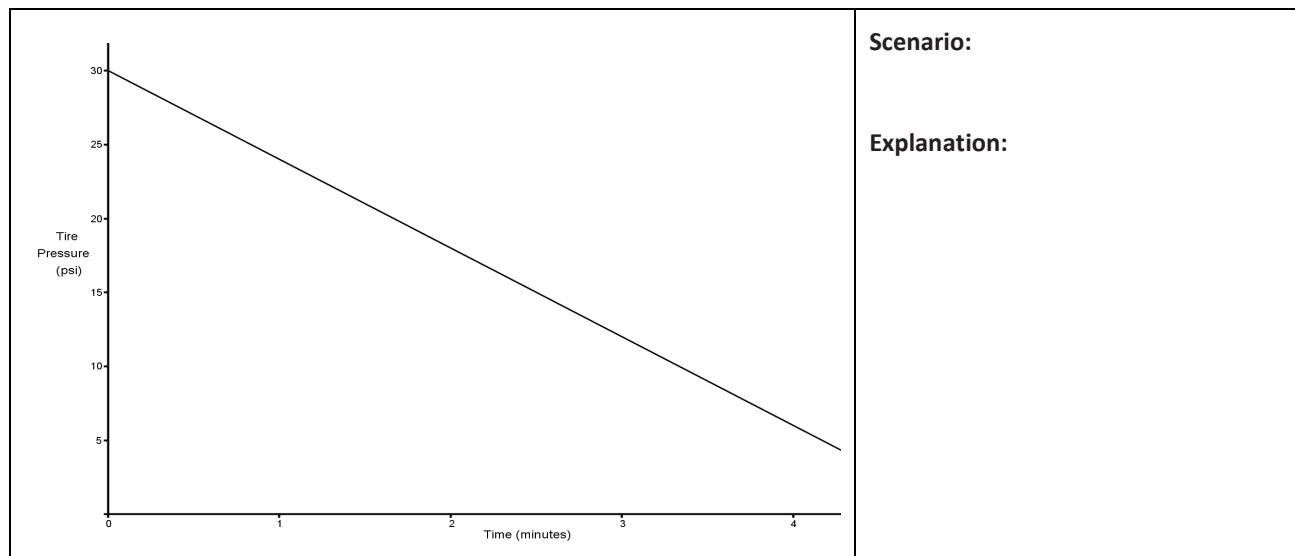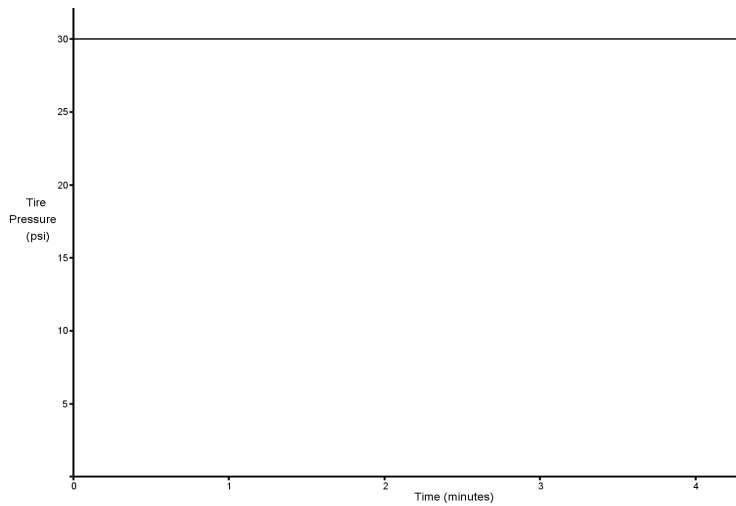
**Explanation:**

**Scenario:**
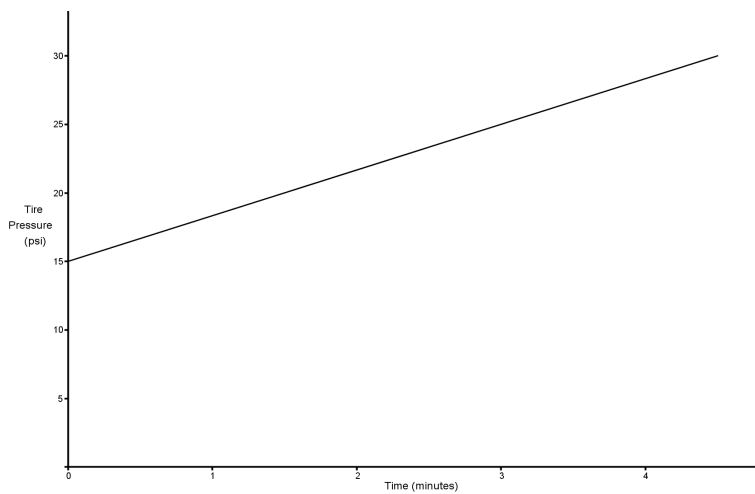
**Explanation:**



**Scenario:**

**Explanation:**

2.  A home was purchased for $275,000.  Due to a recession, the value of the home fell at a constant rate over the next 5 years.

    a.  Sketch a graph of a function that models the situation.
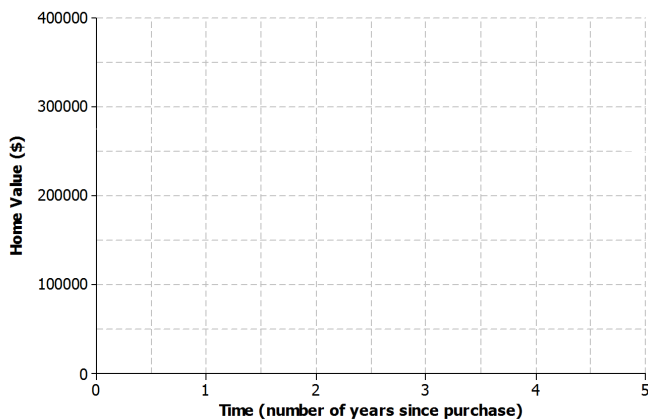


    b.  Based on your graph, how is the home value changing with respect to time?

3.  The graph below displays the first hour of Sam's bike ride.
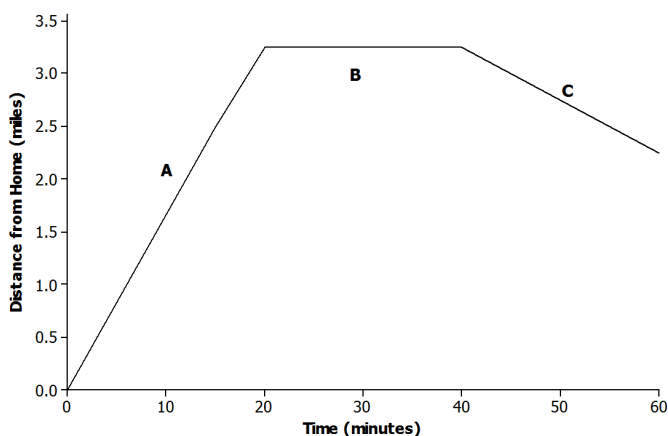


Match each part of the graph (A, B, and C) to its verbal description.  Explain the reasoning behind your choice.

    i.   Sam rides his bike to his friend's house at a constant rate.

    ii.  Sam and his friend bike together to an ice cream shop that is between their houses.

    iii. Sam plays at his friend's house.

4. Using the axes below, create a story about the relationship between two quantities.

   a. Write a story about the relationship between two quantities. Any quantities can be used (e.g., distance and time, money and hours, age and growth). Be creative! Include keywords in your story such as *increase* and *decrease* to describe the relationship.

   b. Label each axis with the quantities of your choice, and sketch a graph of the function that models the relationship described in the story.
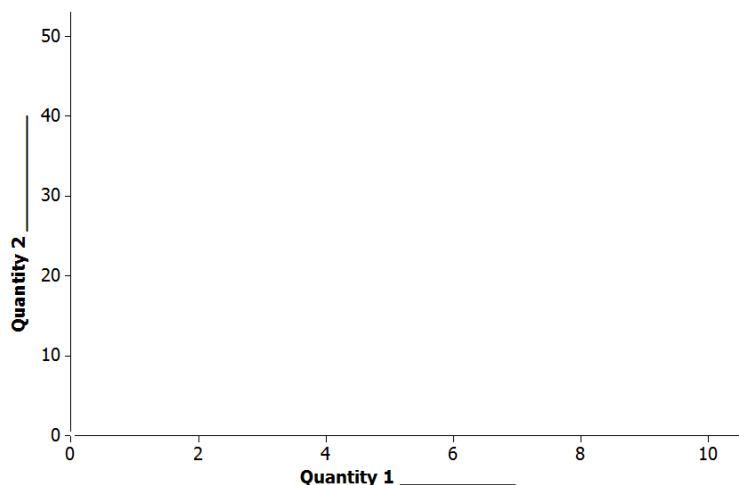
# Lesson 5: Increasing and Decreasing Functions

## Classwork

### Example 1: Nonlinear Functions in the Real World

Not all real-world situations can be modeled by a linear function. There are times when a nonlinear function is needed to describe the relationship between two types of quantities. Compare the two scenarios:

a. Aleph is running at a constant rate on a flat paved road. The graph below represents the total distance he covers with respect to time.
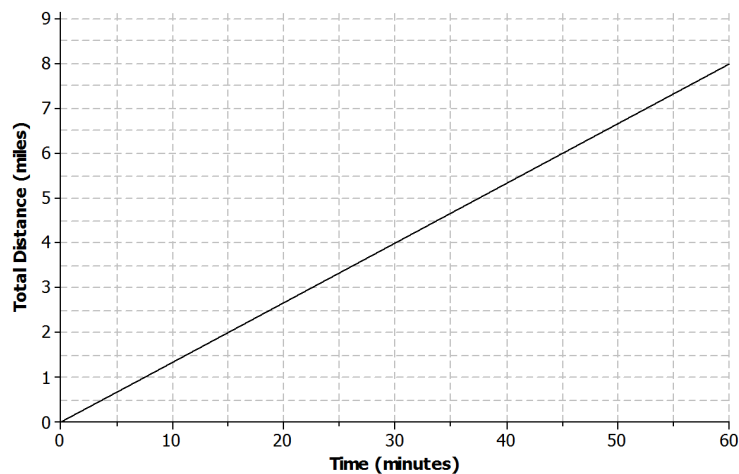


b. Shannon is running on a flat, rocky trail that eventually rises up a steep mountain. The graph below represents the total distance she covers with respect to time.
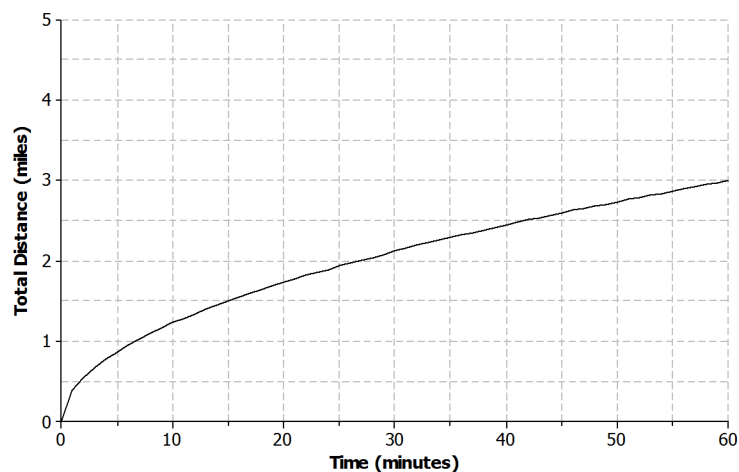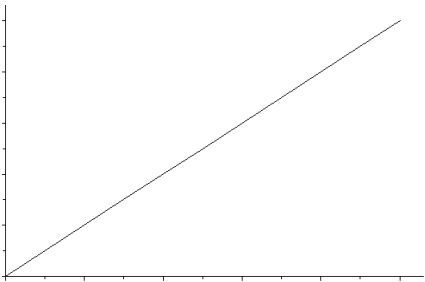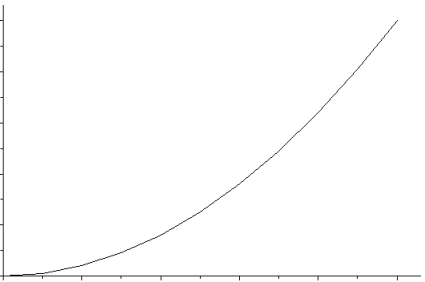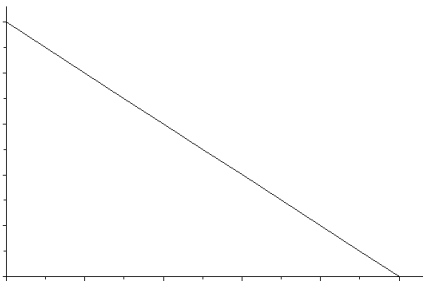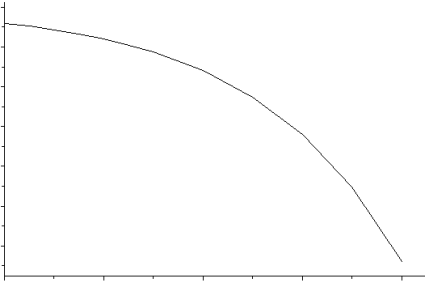
**Exercises 1–2**

1. In your own words, describe what is happening as Aleph is running during the following intervals of time.

    a. 0 to 15 minutes

    b. 15 to 30 minutes

    c. 30 to 45 minutes

    d. 45 to 60 minutes

2. In your own words, describe what is happening as Shannon is running during the following intervals of time.

    a. 0 to 15 minutes

    b. 15 to 30 minutes

    c. 30 to 45 minutes

    d. 45 to 60 minutes

## Example 2: Increasing and Decreasing Functions

The rate of change of a function can provide useful information about the relationship between two quantities. A linear function has a constant rate of change. A nonlinear function has a variable rate of change.

| Linear Functions | Nonlinear Functions |
| --- | --- |
| **Linear function *increasing* at a constant rate** | **Nonlinear function *increasing* at a variable rate** |
|  |  |
| **Linear function *decreasing* at a constant rate** | **Nonlinear function *decreasing* at a variable rate** |
|  |  |
| **Linear function with a constant rate** | **Nonlinear function with a variable rate** |

Linear function with a constant rate:

| $x$ | $y$ |
| --- | --- |
| 0 | 7 |
| 1 | 10 |
| 2 | 13 |
| 3 | 16 |
| 4 | 19 |

Nonlinear function with a variable rate:

| $x$ | $y$ |
| --- | --- |
| 0 | 0 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

**Exercises 3–5**

3. Different breeds of dogs have different growth rates.  A large breed dog typically experiences a rapid growth rate from birth to age 6 months.  At that point, the growth rate begins to slow down until the dog reaches full growth around 2 years of age.

    a. Sketch a graph that represents the weight of a large breed dog from birth to 2 years of age.
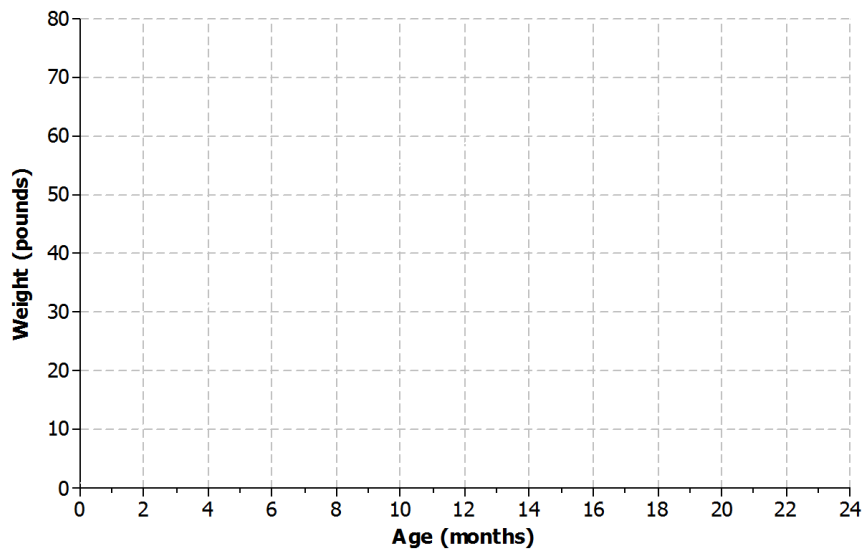


    b. Is the function represented by the graph linear or nonlinear?  Explain.

    c. Is the function represented by the graph increasing or decreasing?  Explain.

4.  Nikka took her laptop to school and drained the battery while typing a research paper.  When she returned home, Nikka connected her laptop to a power source, and the battery recharged at a constant rate.

    a.  Sketch a graph that represents the battery charge with respect to time.
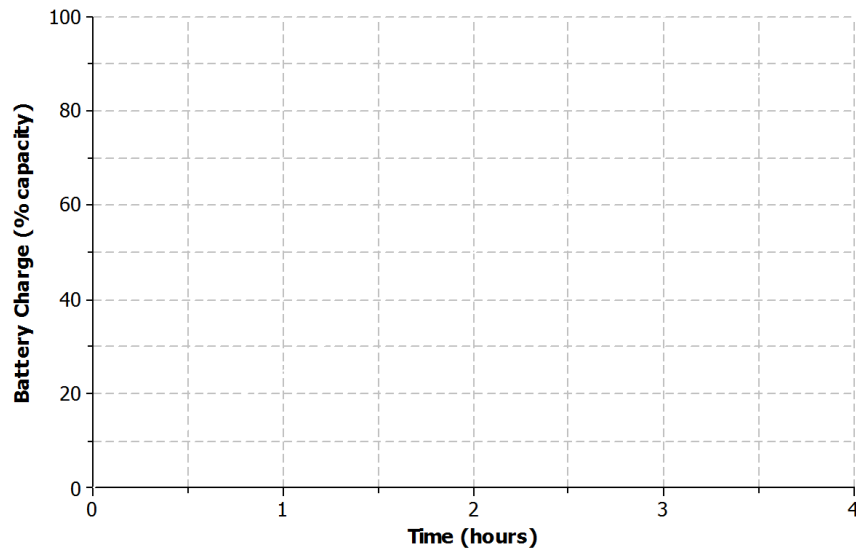


    b.  Is the function represented by the graph linear or nonlinear?  Explain.

    c.  Is the function represented by the graph increasing or decreasing?  Explain.

5.  The long jump is a track and field event where an athlete attempts to leap as far as possible from a given point. Mike Powell of the United States set the long jump world record of 8.95 meters (29.4 feet) during the 1991 World Championships in Tokyo, Japan.

    a.  Sketch a graph that represents the path of a high school athlete attempting the long jump.



    b.  Is the function represented by the graph linear or nonlinear?  Explain.

    c.  Is the function represented by the graph increasing or decreasing?  Explain.

## Example 3:  Ferris Wheel

Lamar and his sister are riding a Ferris wheel at a state fair.  Using their watches, they find that it take 8 seconds for the Ferris wheel to make a complete revolution.  The graph below represents Lamar and his sister's distance above the ground with respect to time.



### Exercises 6–9

6.   Use the graph from Example 3 to answer the following questions.

   a.   Is the function represented by the graph linear or nonlinear?

   b.   Where is the function increasing?  What does this mean within the context of the problem?

   c.   Where is the function decreasing?  What does this mean within the context of the problem?

| Lesson 5: | Increasing and Decreasing Functions |
|-----------|-------------------------------------|
| Date:     | 11/24/14                            |

7.  How high above the ground is the platform for passengers to get on the Ferris wheel? Explain your reasoning.

8.  Based on the graph, how many revolutions does the Ferris wheel complete during the 40 second time interval? Explain your reasoning.

9.  What is the diameter of the Ferris wheel? Explain your reasoning.

---

**Lesson Summary**

The graph of a function can be used to help describe the relationship between two quantities.

A linear function has a constant rate of change. A nonlinear function does not have a constant rate of change.

- A function whose graph has a positive rate of change is an *increasing function*.
- A function whose graph has a negative rate of change is a *decreasing function*.
- Some functions may increase and decrease over different intervals.

---

## Problem Set

1. Read through the following scenarios and match each to its graph. Explain the reasoning behind your choice.
   a. This shows the change in a smartphone battery charge as a person uses the phone more frequently.
   b. A child takes a ride on a swing.
   c. A savings account earns simple interest at a constant rate.
   d. A baseball has been hit at a little league game.
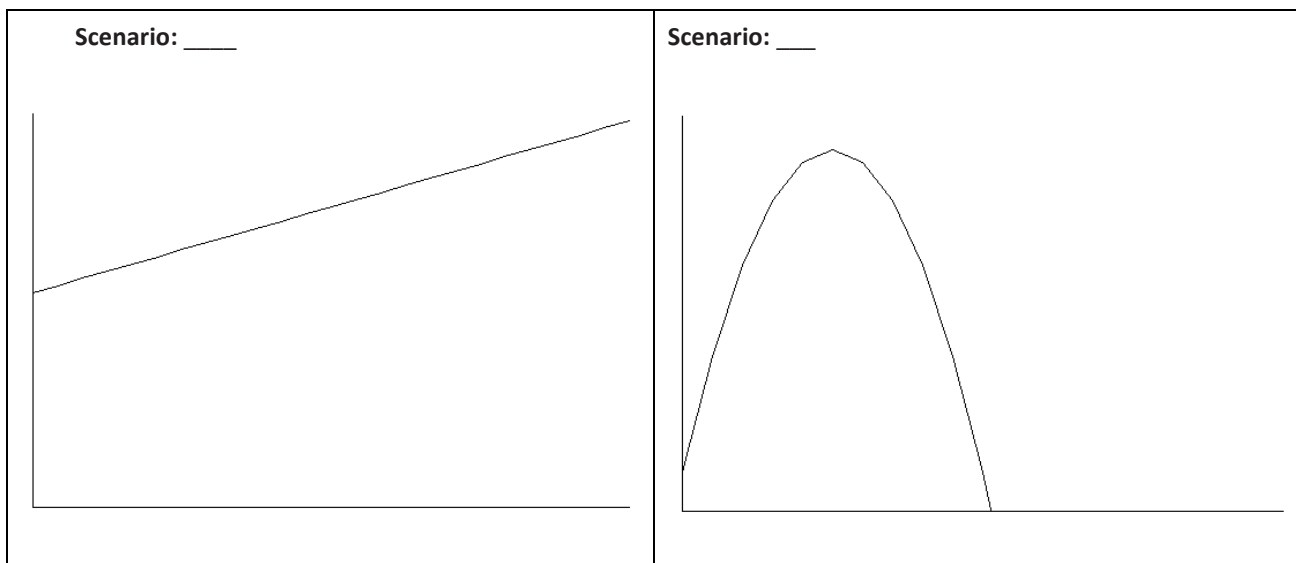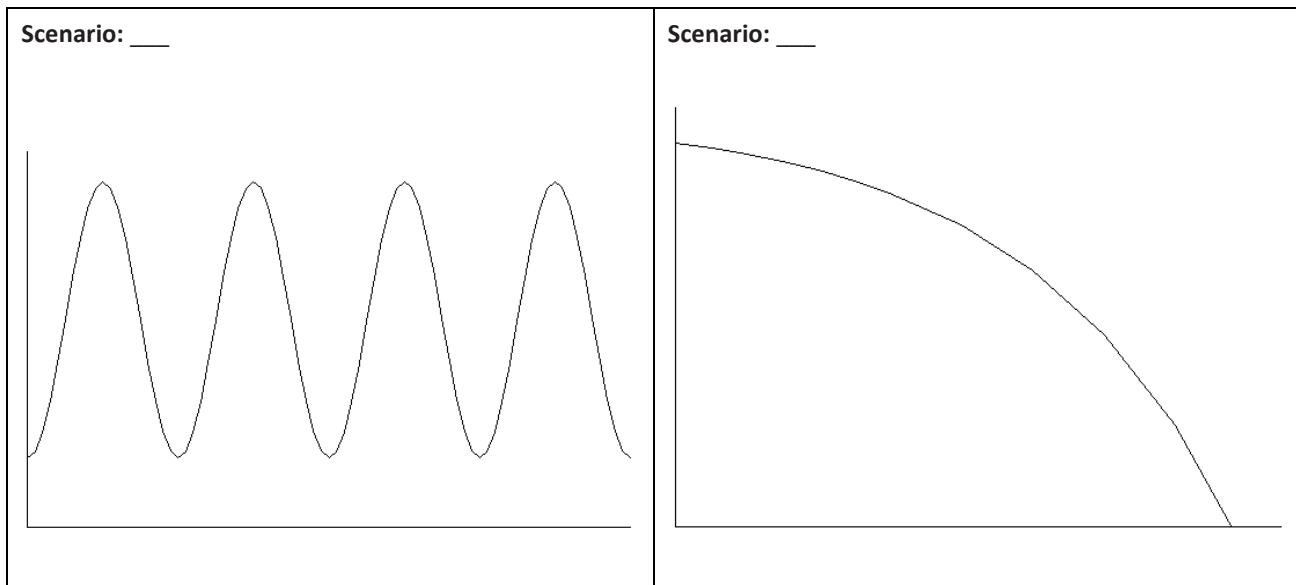
| Scenario: ___ | Scenario: ___ |
|---|---|
|  |  |

2. The graph below shows the volume of water for a given creek bed during a 24-hour period. On this particular day, there was wet weather with a period of heavy rain.
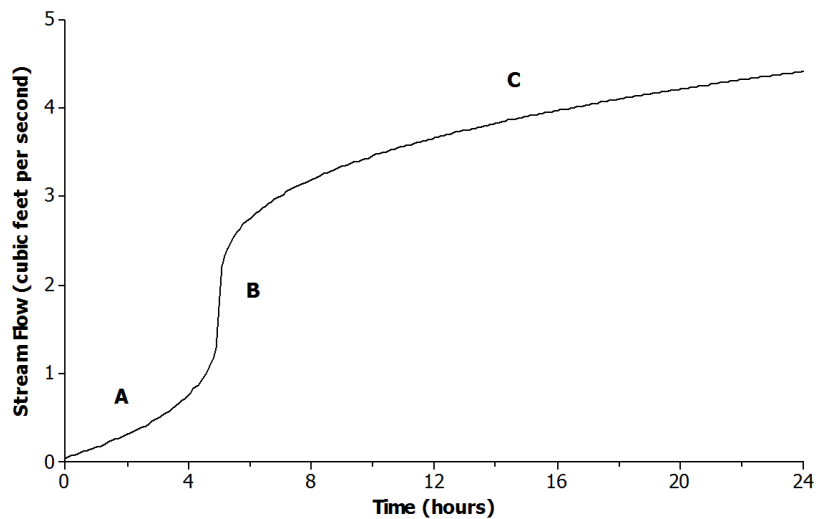


Describe how each part (A, B, and C) of the graph relates to the scenario.

3. Half-life is the time required for a quantity to fall to half of its value measured at the beginning of the time period. If there are 100 grams of a radioactive element to begin with, there will be 50 grams after the first half-life, 25 grams after the second half-life, and so on.

   a. Sketch a graph that represents the amount of the radioactive element left with respect to the number of half-lives that have passed.
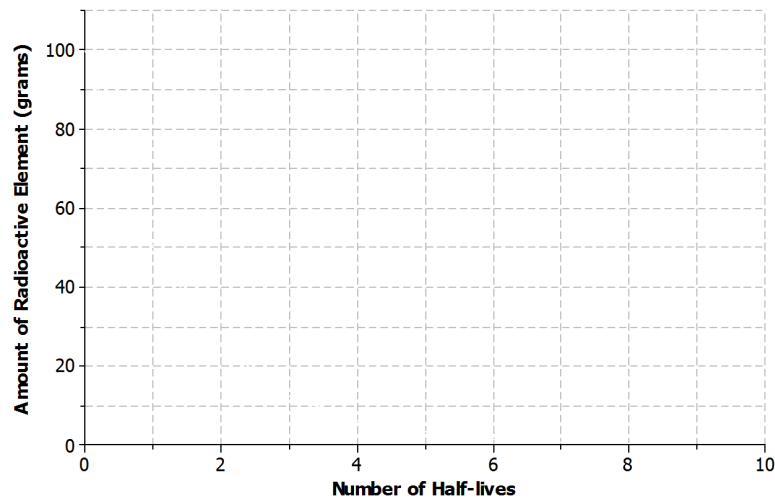


   b. Is the function represented by the graph linear or nonlinear? Explain.

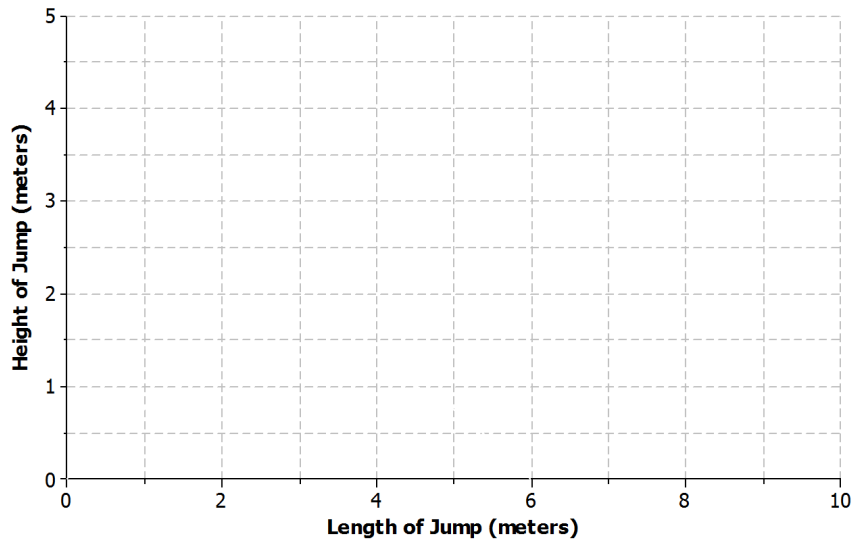   c. Is the function represented by the graph increasing or decreasing?

4. Lanae parked her car in a No Parking zone. Consequently, her car was towed to an impound lot. In order to release her car, she needs to pay the impound lot charges. There is an initial charge on the day the car is brought to the lot. However, 10% of the previous day's charges will be added to the total charge for every day the car remains in the lot.

   a. Sketch a graph that represents the total charges with respect to the number of days a car remains in the impound lot.
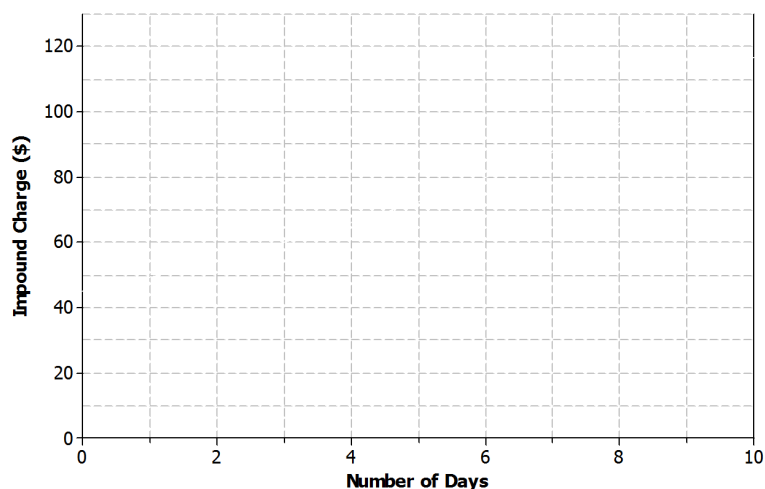
b.  Is the function represented by the graph linear or nonlinear?  Explain.

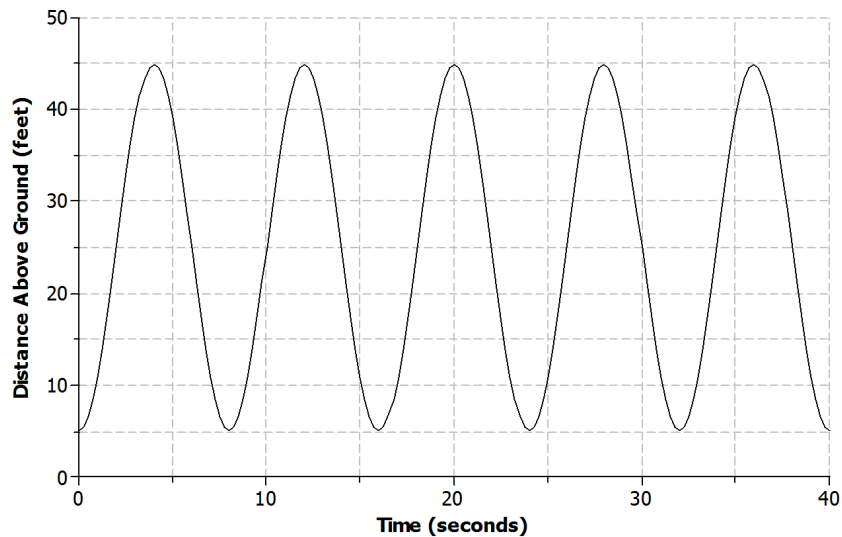c.  Is the function represented by the graph increasing or decreasing?  Explain.

5.  Kern won a $50 gift card to his favorite coffee shop.  Every time he visits the shop, he purchases the same coffee drink.

a.  Sketch a graph of a function that can be used to represent the amount of money that remains on the gift card with respect to the number of drinks purchased.
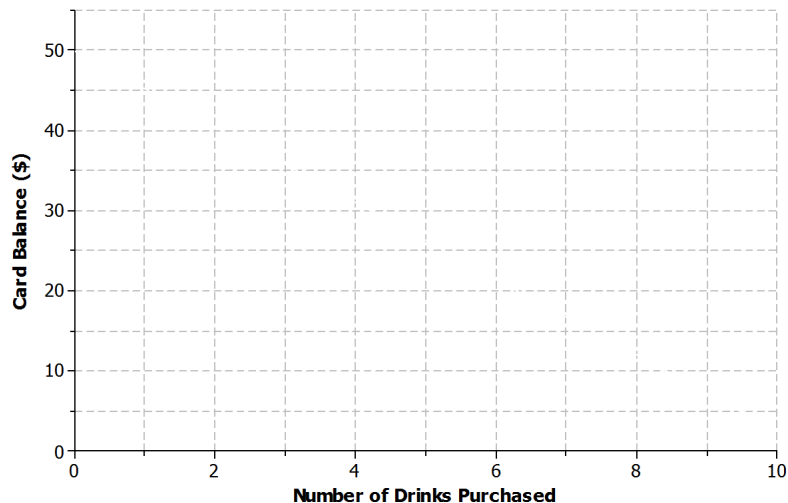


b.  Is the function represented by the graph linear or nonlinear?  Explain.

c.  Is the function represented by the graph increasing or decreasing?  Explain.

6.  Jay and Brooke are racing on bikes to a park 8 miles away.  The tables below display the total distance each person biked with respect to time.

<div style="display:flex; gap:2em;">

**Jay**

| Time (minutes) | Distance (miles) |
|---|---|
| 0 | 0 |
| 5 | 0.84 |
| 10 | 1.86 |
| 15 | 3.00 |
| 20 | 4.27 |
| 25 | 5.67 |

**Brooke**

| Time (minutes) | Distance (miles) |
|---|---|
| 0 | 0 |
| 5 | 1.2 |
| 10 | 2.4 |
| 15 | 3.6 |
| 20 | 4.8 |
| 25 | 6.0 |

</div>

a.  Which person's biking distance could be modeled by a nonlinear function?  Explain.

b.  Who would you expect to win the race?  Explain.

7. Using the axes below, create a story about the relationship between two quantities.

   a. Write a story about the relationship between two quantities. Any quantities can be used (e.g., distance and time, money and hours, age and growth). Be creative! Include keywords in your story such as *increase* and *decrease* to describe the relationship.

   b. Label each axis with the quantities of your choice, and sketch a graph of the function that models the relationship described in the story.
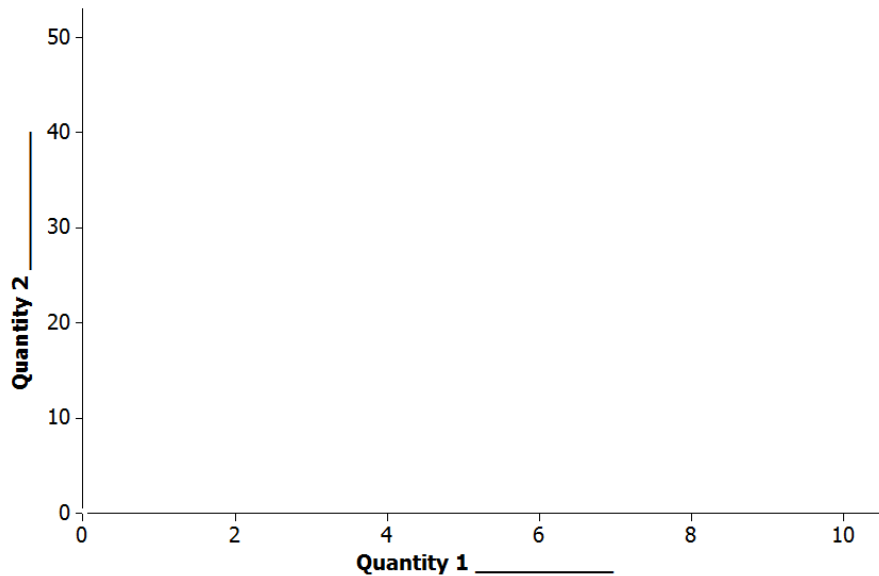
# Lesson 6: Scatter Plots

## Classwork

### Example 1

A bivariate data set consists of observations on two variables. For example, you might collect data on 13 different car models. Each observation in the data set would consist of an $(x, y)$ pair.

$$x = \text{weight (in pounds, rounded to the nearest 50 pounds)}$$

and

$$y = \text{fuel efficiency (in miles per gallon, mpg)}$$

The table below shows the weight and fuel efficiency for 13 car models with automatic transmissions manufactured in 2009 by Chevrolet.

| Model | Weight (pounds) | Fuel Efficiency (mpg) |
|---|---|---|
| 1 | 3,200 | 23 |
| 2 | 2,550 | 28 |
| 3 | 4,050 | 19 |
| 4 | 4,050 | 20 |
| 5 | 3,750 | 20 |
| 6 | 3,550 | 22 |
| 7 | 3,550 | 19 |
| 8 | 3,500 | 25 |
| 9 | 4,600 | 16 |
| 10 | 5,250 | 12 |
| 11 | 5,600 | 16 |
| 12 | 4,500 | 16 |
| 13 | 4,800 | 15 |

### Exercises 1–8

1. In the table above, the observation corresponding to Model 1 is $(3200, 23)$. What is the fuel efficiency of this car? What is the weight of this car?

2. Add the points corresponding to the other 12 observations to the scatter plot.
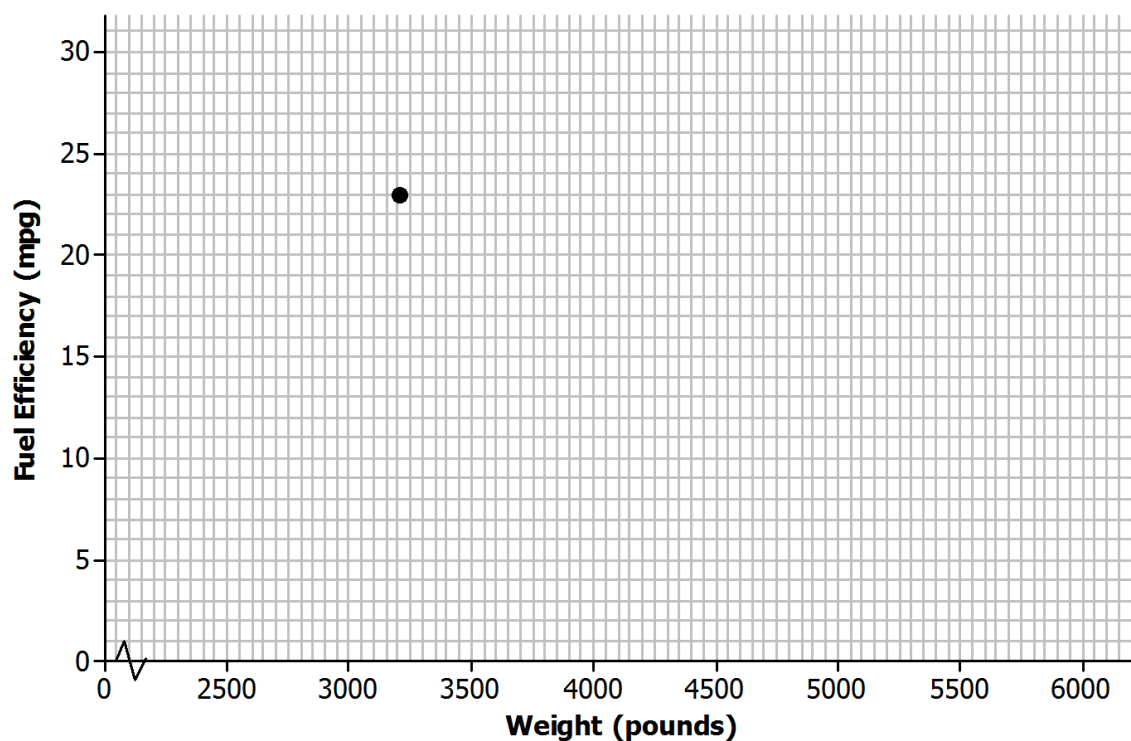


3. Do you notice a pattern in the scatter plot? What does this imply about the relationship between weight ($x$) and fuel efficiency ($y$)?

Is there a relationship between price and the quality of athletic shoes?  The data in the table below are from the Consumer Reports website.

$x$ = price (in dollars)

and

$y$ = Consumer Reports quality rating

The quality rating is on a scale of 0 to 100, with 100 being the highest quality.

| Shoe | Price (dollars) | Quality Rating |
|------|-----------------|----------------|
| 1 | 65 | 71 |
| 2 | 45 | 70 |
| 3 | 45 | 62 |
| 4 | 80 | 59 |
| 5 | 110 | 58 |
| 6 | 110 | 57 |
| 7 | 30 | 56 |
| 8 | 80 | 52 |
| 9 | 110 | 51 |
| 10 | 70 | 51 |

4.  One observation in the data set is $(110, 57)$.  What does this ordered pair represent in terms of cost and quality?

5.  To construct a scatter plot of these data, you need to start by thinking about appropriate scales for the axes of the scatter plot.  The prices in the data set range from \$30 to \$110, so one reasonable choice for the scale of the $x$-axis would range from \$20 to \$120, as shown below.  What would be a reasonable choice for a scale for the $y$-axis?
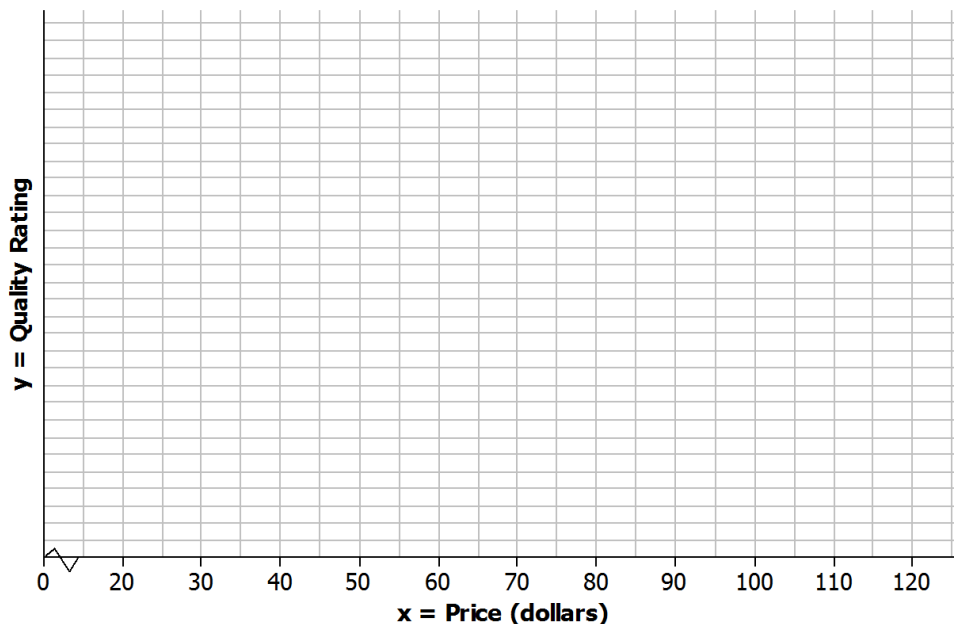
6.   Add a scale to the $y$-axis.  Then, use these axes to construct a scatter plot of the data.

7.   Do you see any pattern in the scatter plot indicating that there is a relationship between price and quality rating for athletic shoes?

8.   Some people think that if shoes have a high price, they must be of high quality.  How would you respond?

### Example 2:  Statistical Relationships

A pattern in a scatter plot indicates that the values of one variable tend to vary in a predictable way as the values of the other variable change.  This is called a *statistical relationship*.  In the fuel efficiency and car weight example, fuel efficiency tended to decrease as car weight increased.

This is useful information, but be careful not to jump to the conclusion that increasing the weight of a car *causes* the fuel efficiency to go down.  There may be some other explanation for this.  For example, heavier cars may also have bigger engines, and bigger engines may be less efficient.  You cannot conclude that changes to one variable *cause* changes in the other variable just because there is a statistical relationship in a scatter plot.

**Exercises 9–10**

9.  Data were collected on

$$x = \text{shoe size}$$

and

$$y = \text{score on a reading-ability test}$$

for 30 elementary school students.  The scatter plot of these data is shown below.  Does there appear to be a statistical relationship between shoe size and score on the reading test?
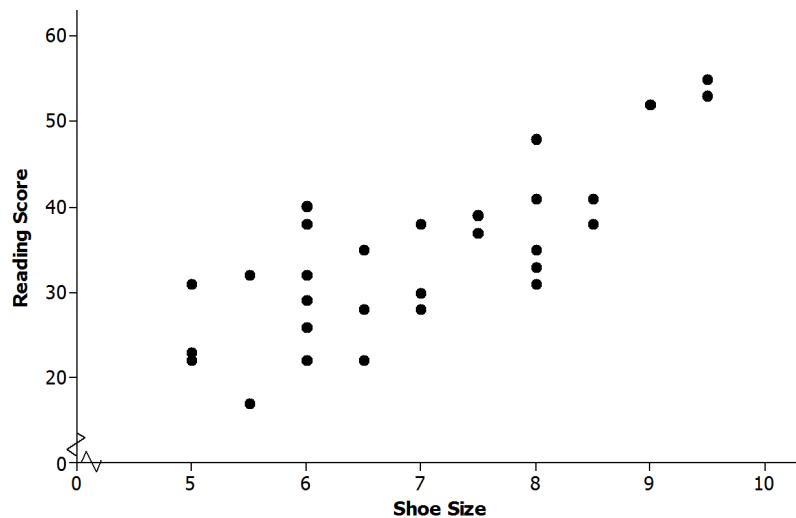


10. Explain why it is not reasonable to conclude that having big feet causes a high reading score.  Can you think of a different explanation for why you might see a pattern like this?

**Lesson Summary**

- A scatter plot is a graph of numerical data on two variables.

- A pattern in a scatter plot suggests that there may be a relationship between the two variables used to construct the scatter plot.

- If two variables tend to vary together in a predictable way, we can say that there is a statistical relationship between the two variables.

- A statistical relationship between two variables does not imply that a change in one variable causes a change in the other variable (a cause-and-effect relationship).

## Problem Set

1. The table below shows the price and overall quality rating for 15 different brands of bike helmets.

   Data Source: www.consumerreports.org

| Helmet | Price (dollars) | Quality Rating |
|--------|-----------------|----------------|
| A | 35 | 65 |
| B | 20 | 61 |
| C | 30 | 60 |
| D | 40 | 55 |
| E | 50 | 54 |
| F | 23 | 47 |
| G | 30 | 47 |
| H | 18 | 43 |
| I | 40 | 42 |
| J | 28 | 41 |
| K | 20 | 40 |
| L | 25 | 32 |
| M | 30 | 63 |
| N | 30 | 63 |
| O | 40 | 53 |

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Lesson 6   8•6

Construct a scatter plot of price ($x$) and quality rating ($y$).  Use the grid below.
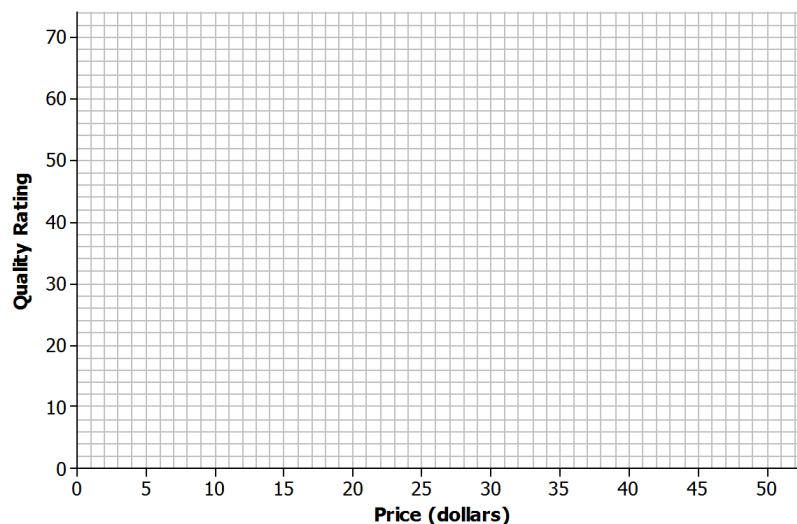


2.  Do you think that there is a statistical relationship between price and quality rating?  If so, describe the nature of the relationship.

3.  Scientists are interested in finding out how different species adapt to finding food sources.  One group studied crocodilian species to find out how their bite force was related to body mass and diet.  The table below displays the information they collected on body mass (in pounds) and bite force (in pounds).

| Species | Body Mass (pounds) | Bite Force (pounds) |
|---|---|---|
| Dwarf crocodile | 35 | 450 |
| Crocodile F | 40 | 260 |
| Alligator A | 30 | 250 |
| Caiman A | 28 | 230 |
| Caiman B | 37 | 240 |
| Caiman C | 45 | 255 |
| Croc A | 110 | 550 |
| Nile crocodile | 275 | 650 |
| Croc B | 130 | 500 |
| Croc C | 135 | 600 |
| Croc D | 135 | 750 |
| Caiman D | 125 | 550 |
| Indian Gharial croc | 225 | 400 |
| Crocodile G | 220 | 1,000 |
| American croc | 270 | 900 |
| Croc D | 285 | 750 |
| Croc E | 425 | 1,650 |
| American Alligator | 300 | 1,150 |
| Alligator B | 325 | 1,200 |
| Alligator C | 365 | 1,450 |

Data Source:  PLoS One Greg Erickson biomechanics, Florida State University

**COMMON CORE**

Lesson 6:     Scatter Plots
Date:          11/24/14

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Construct a scatter plot of body mass ($x$) and bite force ($y$). Use the grid below, and be sure to add an appropriate scale to the axes.
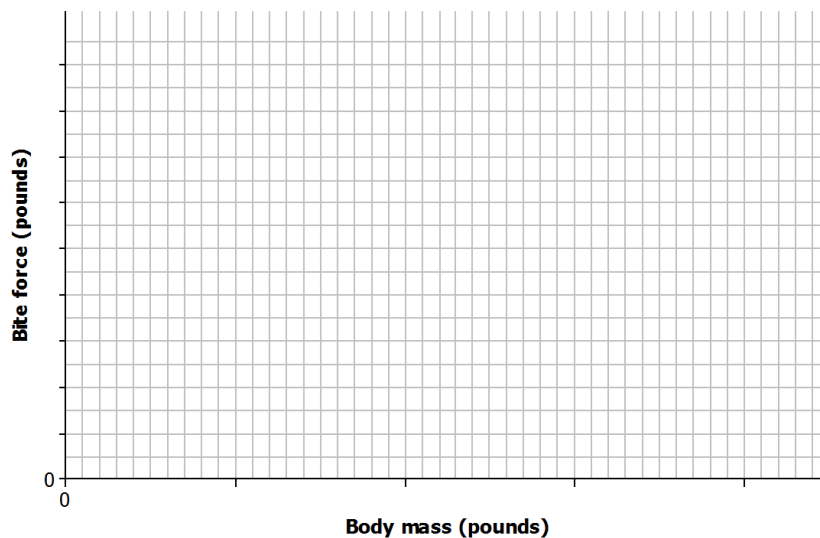


4. Do you think that there is a statistical relationship between body mass and bite force? If so, describe the nature of the relationship.

5. Based on the scatter plot, can you conclude that increased body mass causes increased bite force? Explain.

# Lesson 7:  Patterns in Scatter Plots

### Example 1

In the previous lesson, you learned that scatterplots show trends in bivariate data.

When you look at a scatter plot, you should ask yourself the following questions:

a. Does it look like there is a relationship between the two variables used to make the scatter plot?

b. If there is a relationship, does it appear to be linear?

c. If the relationship appears to be linear, is the relationship a positive linear relationship or a negative linear relationship?

To answer the first question, look for patterns in the scatter plot.  Does there appear to be a general pattern to the points in the scatter plot, or do the points look as if they are scattered at random?  If you see a pattern, you can answer the second question by thinking about whether the pattern would be well-described by a line.  Answering the third question requires you to distinguish between a positive linear relationship and a negative linear relationship.  A positive linear relationship is one that is described by a line with a positive slope.  A negative linear relationship is one that is described by a line with a negative slope.

**Exercises 1–9**

Take a look at the following five scatter plots.  Answer the three questions above for each scatter plot.

1. Scatter Plot 1
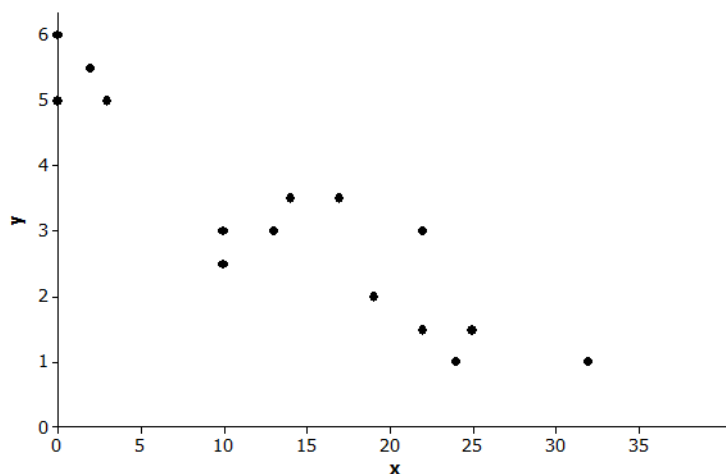


Is there a relationship?

If there is a relationship, does it appear to be linear?

If the relationship appears to be linear, is it a positive or negative linear relationship?

2.  Scatter Plot 2
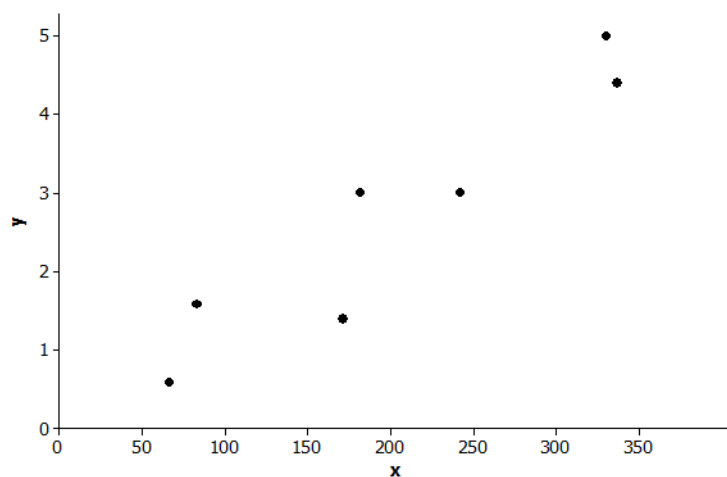


Is there a relationship?

If there is a relationship, does it appear to be linear?

If the relationship appears to be linear, is it a positive or negative linear relationship?

3.  Scatter Plot 3
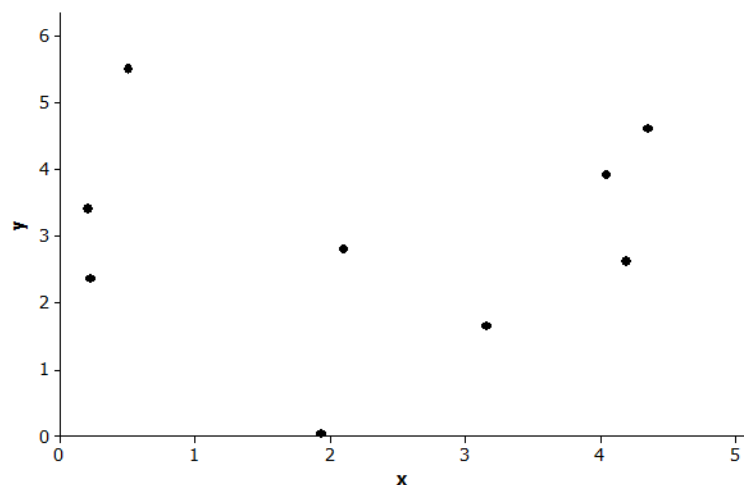


Is there a relationship?

If there is a relationship, does it appear to be linear?

If the relationship appears to be linear, is it a positive or negative linear relationship?

4.   Scatter Plot 4
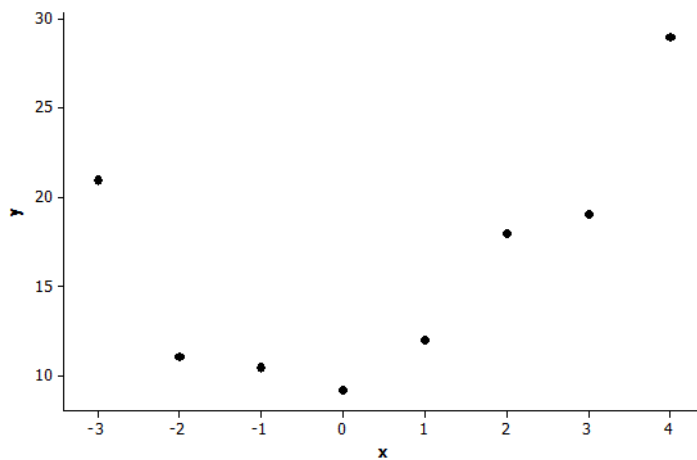


Is there a relationship?

If there is a relationship, does it appear to be linear?

If the relationship appears to be linear, is it a positive or negative linear relationship?

5.   Scatter Plot 5
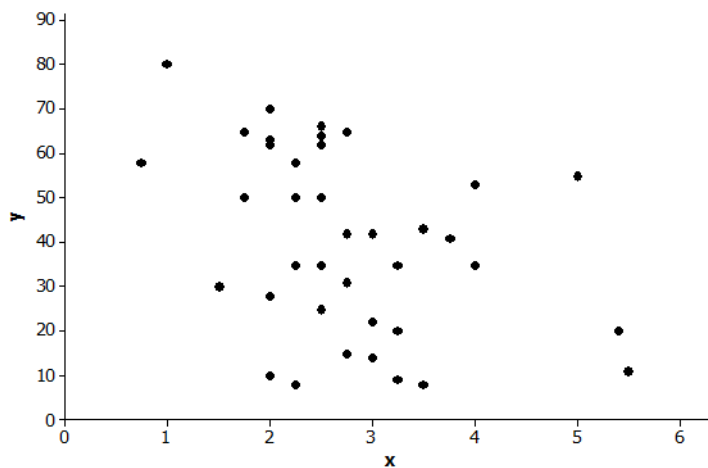


Is there a relationship?

If there is a relationship, does it appear to be linear?

If the relationship appears to be linear, is it a positive or negative linear relationship?

6.  Below is a scatter plot of data on weight in pounds ($x$) and fuel efficiency in miles per gallon ($y$) for 13 cars.  Using the questions at the beginning of this lesson as a guide, write a few sentences describing any possible relationship between $x$ and $y$.
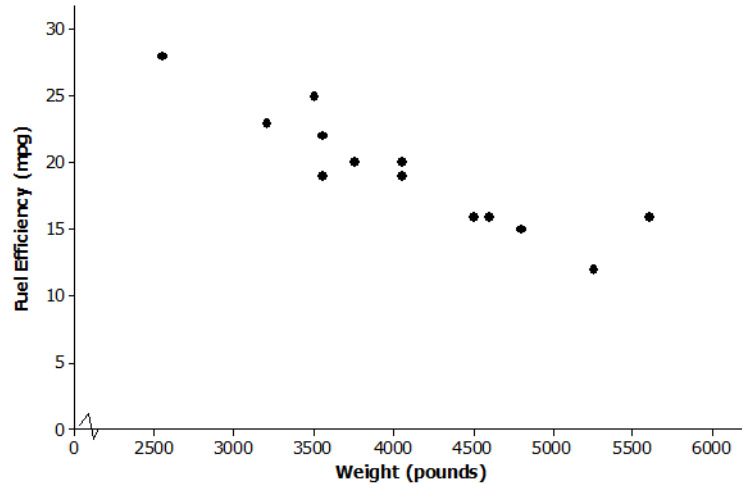


7.  Below is a scatter plot of data on price in dollars ($x$) and quality rating ($y$) for 14 bike helmets.  Using the questions at the beginning of this lesson as a guide, write a few sentences describing any possible relationship between $x$ and $y$.
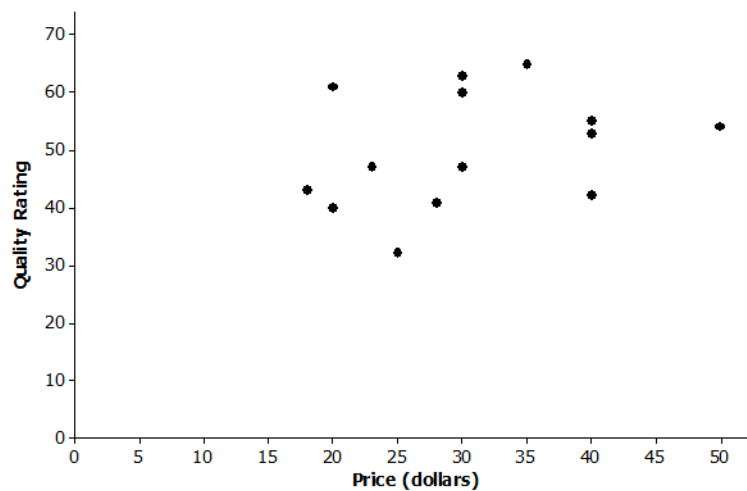
8.  Below is a scatter plot of data on shell length in millimeters ($x$) and age in years ($y$) for 27 lobsters of known age. Using the questions at the beginning of this lesson as a guide, write a few sentences describing any possible relationship between $x$ and $y$.
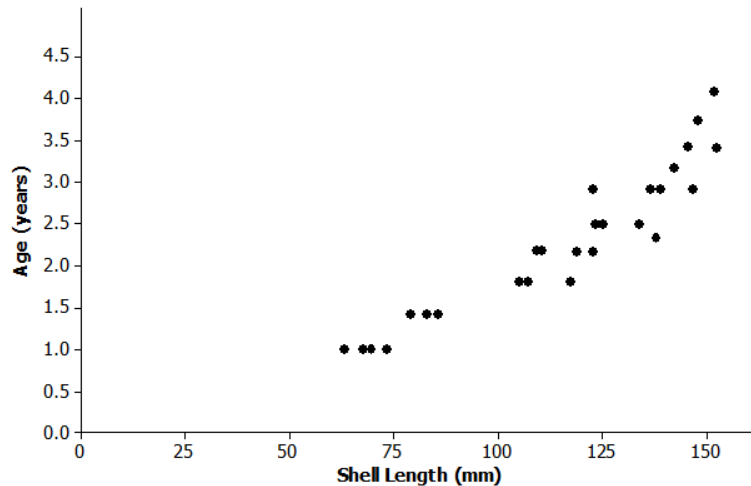


9.  Below is a scatter plot of data from crocodiles on body mass in pounds ($x$) and bite force in pounds ($y$). Using the questions at the beginning of this lesson as a guide, write a few sentences describing any possible relationship between $x$ and $y$.
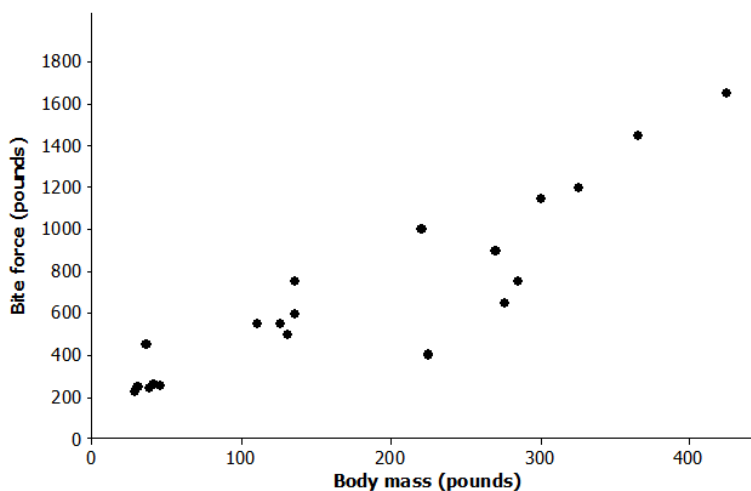
## Example 2: Clusters and Outliers

In addition to looking for a general pattern in a scatter plot, you should also look for other interesting features that might help you understand the relationship between two variables. Two things to watch for are as follows:

- **CLUSTERS:** Usually the points in a scatter plot form a single cloud of points, but sometimes the points may form two or more distinct clouds of points. These clouds are called *clusters*. Investigating these clusters may tell you something useful about the data.

- **OUTLIERS:** An *outlier* is an unusual point in a scatter plot that does not seem to fit the general pattern or that is far away from the other points in the scatter plot.

The scatter plot below was constructed using data from a study of Rocky Mountain elk ("Estimating Elk Weight from Chest Girth," *Wildlife Society Bulletin*, 1996). The variables studied were chest girth in centimeter ($x$) and weight in kilogram ($y$).
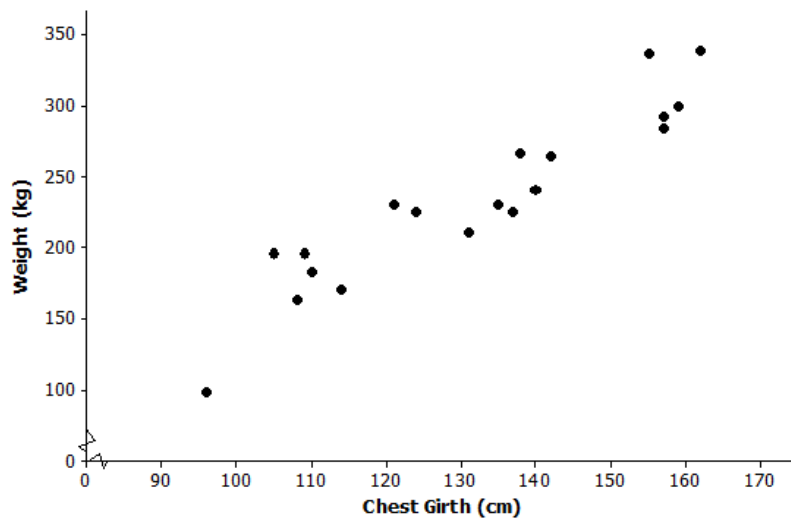


### Exercises 10–12

10. Do you notice any point in the scatter plot of elk weight versus chest girth that might be described as an outlier? If so, which one?

11. If you identified an outlier in Exercise 10, write a sentence describing how this data observation differs from the others in the data set.

12. Do you notice any clusters in the scatter plot?  If so, how would you distinguish between the clusters in terms of chest girth?  Can you think of a reason these clusters might have occurred?

**Lesson Summary**

- A scatter plot might show a linear relationship, a nonlinear relationship, or no relationship.

- A positive linear relationship is one that would be modeled using a line with a positive slope. A negative linear relationship is one that would be modeled by a line with a negative slope.

- Outliers in a scatter plot are unusual points that do not seem to fit the general pattern in the plot or that are far away from the other points in the scatter plot.

- Clusters occur when the points in the scatter plot appear to form two or more distinct clouds of points.

## Problem Set

1. The scatter plot below was constructed using data size in square feet ($x$) of several houses and price in dollars ($y$). Write a few sentences describing the relationship between price and size for these houses. Are there any noticeable clusters or outliers?
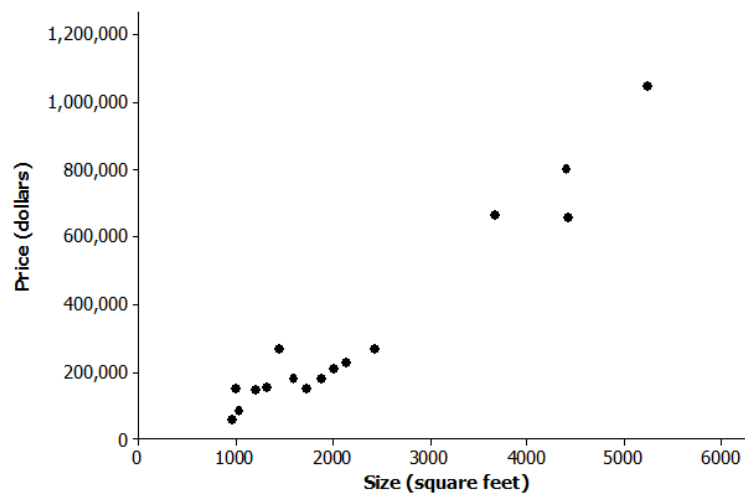
2. The scatter plot below was constructed using data on length in inches ($x$) of several alligators and weight in pounds ($y$). Write a few sentences describing the relationship between weight and length for these alligators. Are there any noticeable clusters or outliers?
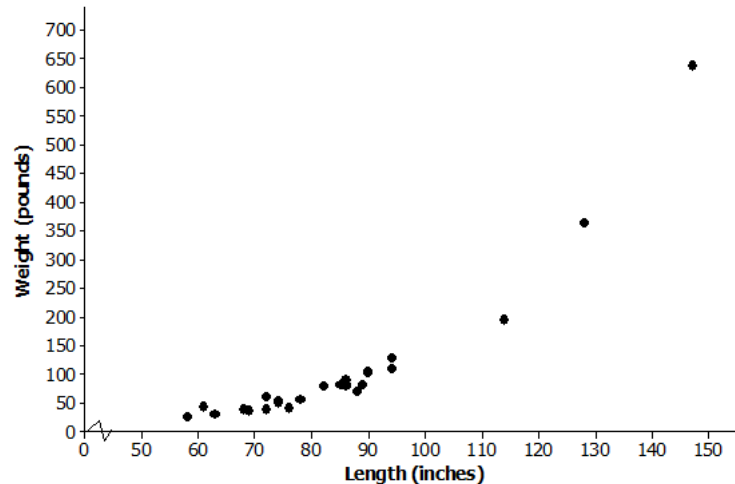


3. The scatter plot below was constructed using data on age in years ($x$) of several Honda Civics and price in dollars ($y$). Write a few sentences describing the relationship between price and age for these cars. Are there any noticeable clusters or outliers?
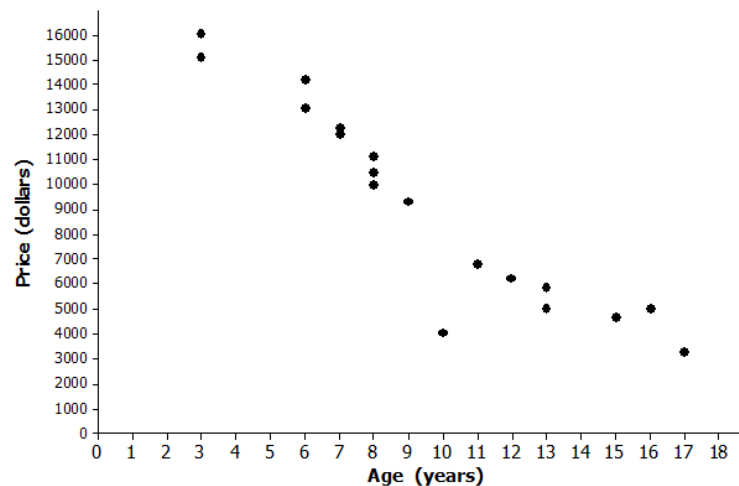
4. Samples of students in each of the U.S. states periodically take part in a large-scale assessment called the National Assessment of Educational Progress (NAEP).  The table below shows the percent of students in the northeastern states (as defined by the U.S. Census Bureau) who answered problems 7 and 15 correctly on the 2011 eighth-grade test.  The scatter plot shows the percent of eighth-grade students who got problems 7 and 15 correct on the 2011 NAEP.

| State | % Correct Problem 7 | % Correct Problem 15 |
|---|---|---|
| Connecticut | 29 | 51 |
| New York | 28 | 47 |
| Rhode Island | 29 | 52 |
| Maine | 27 | 50 |
| Pennsylvania | 29 | 48 |
| Vermont | 32 | 58 |
| New Jersey | 35 | 54 |
| New Hampshire | 29 | 52 |
| Massachusetts | 35 | 56 |

**Percent Correct for Problems 7 and 15 on 2011 Eighth-Grade NAEP**
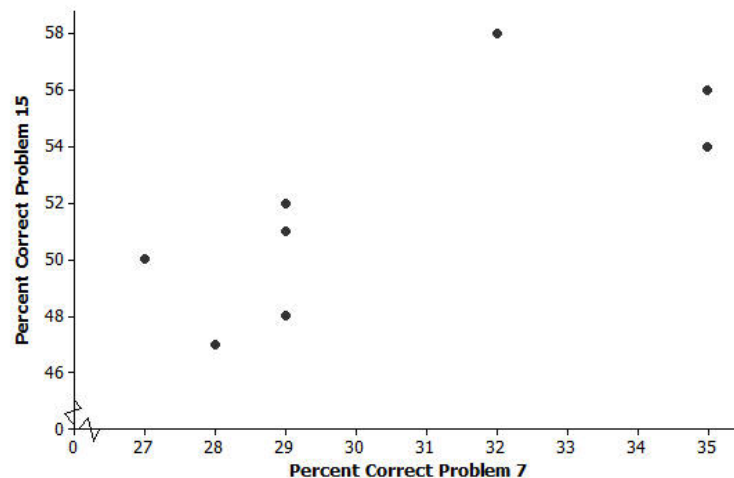


a. Why does it appear that there are only eight points in the scatter plot for nine states?

b. What is true of the states represented by the cluster of five points in the lower left corner of the graph?

c. Which state did the best on these two problems?  Explain your reasoning.

d. Is there a trend in the data?  Explain your thinking.

5.  The plot below shows the mean percent of sunshine during the year and the mean amount of precipitation in inches per year for the states in the United States.
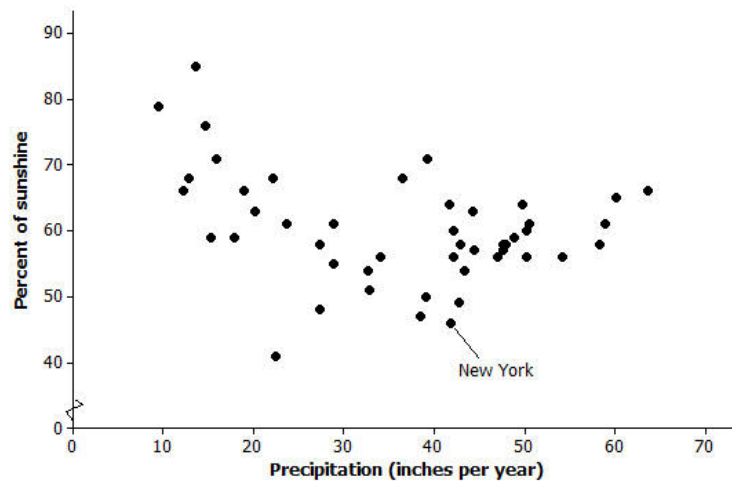


Data Source:    www.currentresults.com/Weather/US/average-annual-state-sunshine.php
www.currentresults.com/Weather/US/average-annual-state-precipitation.php

a.  Where on the graph are the states that have a large amount of precipitation and a small percent of sunshine?

b.  The state of New York is the point $(46, 41.8)$.  Describe how the mean amount of precipitation and percent of sunshine in New York compare to the rest of the United States.

c.  Write a few sentences describing the relationship between mean amount of precipitation and percent of sunshine.

6.  At a dinner party, every person shakes hands with every other person present.

a.  If three people are in a room and everyone shakes hands with everyone else, how many handshakes will there be?

b.  Make a table for the number of handshakes in the room for one to six people.  You may want to make a diagram or list to help you count the number of handshakes.

| Number People | Handshakes | | Number People | Handshakes |
|---|---|---|---|---|
|  |  | |  |  |
|  |  | |  |  |
|  |  | |  |  |

c.    Make a scatter plot of number of people ($x$) and number of handshakes ($y$).  Explain your thinking.
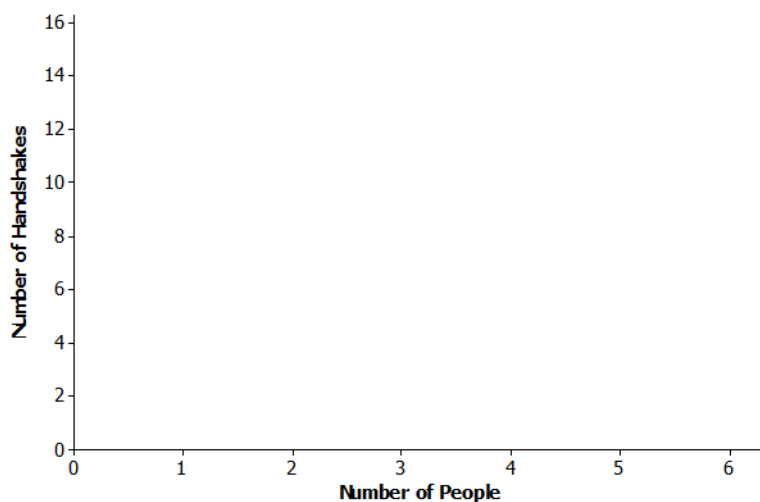


d.    Does the trend seem to be linear?  Why or why not?

# Lesson 8: Informally Fitting a Line

## Classwork

### Example 1: Housing Costs

Let's look at some data from one Midwestern city that indicates the sizes and sale prices of various houses sold in this city.

| Size (square feet) | Price (dollars) | Size (square feet) | Price (dollars) |
|---|---|---|---|
| 5,232 | 1,050,000 | 1,196 | 144,900 |
| 1,875 | 179,900 | 1,719 | 149,900 |
| 1,031 | 84,900 | 956 | 59,900 |
| 1,437 | 269,900 | 991 | 149,900 |
| 4,400 | 799,900 | 1,312 | 154,900 |
| 2,000 | 209,900 | 4,417 | 659,999 |
| 2,132 | 224,900 | 3,664 | 669,000 |
| 1,591 | 179,900 | 2,421 | 269,900 |

Data Source: http://www.trulia.com/for_sale/Milwaukee,WI/5_p

A scatter plot of the data is given below.

**Exercises 1–6**

1.  What can you tell about the price of large homes compared to the price of small homes from the table?

2.  Use the scatter plot to answer the following questions.

    a.  Does the scatter plot seem to support the statement that larger houses tend to cost more?  Explain your thinking.

    b.  What is the cost of the most expensive house, and where is that point on the scatter plot?

    c.  Some people might consider a given amount of money and then predict what size house they could buy.  Others might consider what size house they want and then predict how much it would cost.  How would you use the above scatter plot?

    d.  Estimate the cost of a 3,000 square foot house.

    e.  Do you think a line would provide a reasonable way to describe how price and size are related?  How could you use a line to predict the price of a house if you are given its size?

3.   Draw a line in the plot that you think would fit the trend in the data.

4.   Use your line to answer the following questions:

   a.   What is your prediction of the price of a 3,000 square foot house?

   b.   What is the prediction of the price of a 1,500 square foot house?

5.   Consider the following general strategies students use for drawing a line.  Do you think they represent a good strategy for drawing a line that will fit the data?  Explain why or why not, or draw a line for the scatter plot using the strategy that would indicate why it is or why it is not a good strategy.

   a.   Laure thought she might draw her line using the very first point (farthest to the left) and the very last point (farthest to the right) in the scatter plot.

   b.   Phil wants to be sure that he has the same number of points above and below the line.

   c.   Sandie thought she might try to get a line that had the most points right on it.

   d.   Maree decided to get her line as close to as many of the points as possible.

6.   Based on the strategies discussed in Exercise 5, would you change how you draw a line through the points?  Explain your answer.

## Example 2:  Deep Water

Does the current in the water go faster or slower when the water is shallow?  The data on the depth and speed of the Columbia River at various locations in Washington state listed below can help you think about the answer.

**Depth and Velocity in the Columbia River, Washington State**

| Depth (feet) | Velocity (feet/second) |
|:---:|:---:|
| 0.7 | 1.55 |
| 2.0 | 1.11 |
| 2.6 | 1.42 |
| 3.3 | 1.39 |
| 4.6 | 1.39 |
| 5.9 | 1.14 |
| 7.3 | 0.91 |
| 8.6 | 0.59 |
| 9.9 | 0.59 |
| 10.6 | 0.41 |
| 11.2 | 0.22 |

Data Source:  www.seattlecentral.edu/qelp/sets/011/011.html

a.   What can you tell about the relationship between the depth and velocity by looking at the numbers in the table?

b.   If you were to make a scatter plot of the data, which variable would you put on the horizontal axis and why?

**Exercises 7–9**

7.  A scatter plot of the Columbia River data is shown below.
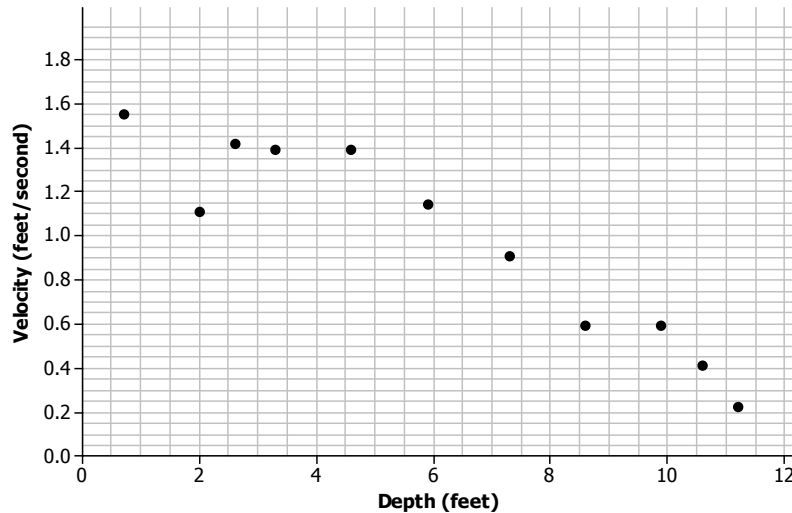


a.  Choose a data point in the scatter plot and describe what it means in terms of the context.

b.  Based on the scatter plot, describe the relationship between velocity and depth.

c.  How would you explain the relationship between the velocity and depth of the water?

d.  If the river is two feet deep at a certain spot, how fast do you think the current would be?  Explain your reasoning.

8.   Consider the following questions:

    a.   If you draw a line to represent the trend in the plot, will it make it easier to predict the velocity of the water if you know the depth?  Why or why not?

    b.   Draw a line that you think does a reasonable job of modeling the trend on the scatter plot above.  Use the line to predict the velocity when the water is 8 feet deep.

9.   Use the line to predict the velocity for a depth of 8.6 feet.  How far off was your prediction from the actual observed velocity for the location that had a depth of 8.6 feet?

**Lesson Summary**

- When constructing a scatter plot, the variable that you want to predict (i.e., the dependent or response variable) goes on the vertical axis. The independent variable (i.e., the variable not changed by other variables) goes on the horizontal axis.

- When the pattern in a scatter plot is approximately linear, a line can be used to describe the linear relationship.

- A line that describes the relationship between a dependent variable and an independent variable can be used to make predictions of the value of the dependent variable given a value of the independent variable.

- When informally fitting a line, you want to find a line for which the points in the scatter plot tend to be closest.

## Problem Set

1. The table below shows the mean temperature in July and the mean amount of rainfall per year for 14 cities in the Midwest.

| City | Mean Temperature in July (Degrees Fahrenheit) | Mean Rainfall per Year (inches) |
|---|---|---|
| Chicago, IL | 73.3 | 36.27 |
| Cleveland, OH | 71.9 | 38.71 |
| Columbus, OH | 75.1 | 38.52 |
| Des Moines, IA | 76.1 | 34.72 |
| Detroit, MI | 73.5 | 32.89 |
| Duluth, MN | 65.5 | 31.00 |
| Grand Rapids, MI | 71.4 | 37.13 |
| Indianapolis, IN | 75.4 | 40.95 |
| Marquette, MI | 71.6 | 32.95 |
| Milwaukee, WI | 72.0 | 34.81 |
| Minneapolis–St. Paul, MN | 73.2 | 29.41 |
| Springfield, MO | 76.3 | 35.56 |
| St. Louis, MO | 80.2 | 38.75 |
| Rapid City, SD | 73.0 | 33.21 |

Data Source: http://countrystudies.us/united-states/weather/

a. What do you observe from looking at the data in the table?

b.   Look at the scatter plot below.  A line is drawn to fit the data.  The plot in the Exit Ticket had the mean July temperatures for the cities on the horizontal axis.  How is this plot different, and what does it mean for the way you think about the relationship between the two variables, temperature and rain?

**July Rainfall and Temperatures in Selected Midwestern Cities**
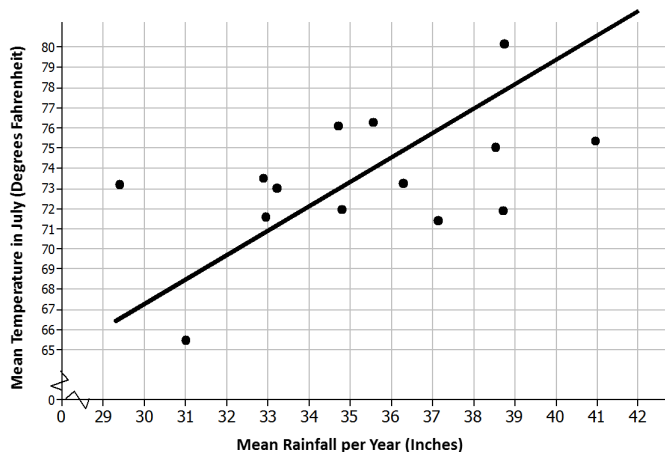


c.   The line has been drawn to model the relationship between the amount of rain and the temperature in those Midwestern cities.  Use the line to predict the mean July temperature for a Midwestern city that has a mean of 32 inches of rain per year.

d.   For which of the cities in the sample will the line do the worst job of predicting the mean temperature?  The best?  Explain your reasoning with as much detail as possible.

2.   The scatter plot below shows the results of a survey of eighth-grade students who were asked to report the number of hours per week they spend playing video games and the typical number of hours they sleep each night.

**Mean Hours Sleep per Night vs. Mean Hours Playing Video Games per Week**
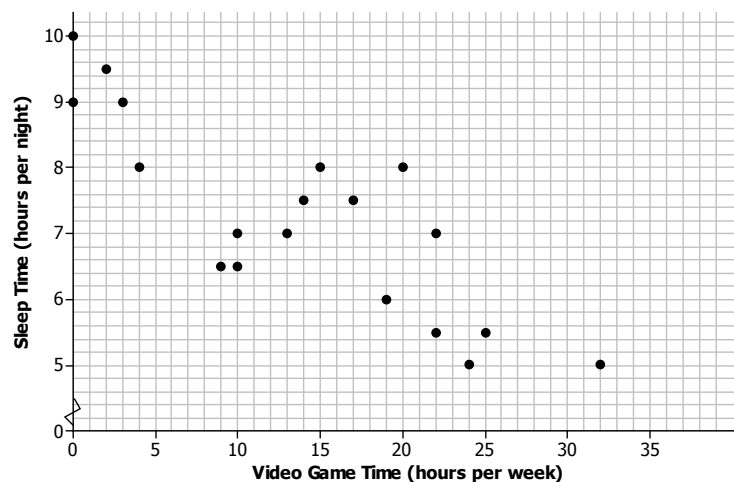
   a.   What trend do you observe in the data?

   b.   What was the fewest number of hours per week that students who were surveyed spent playing video games? The most?

   c.   What was the fewest number of hours per night that students who were surveyed typically slept? The most?

   d.   Draw a line that seems to fit the trend in the data and find its equation. Use the line to predict the number of hours of sleep for a student who spends about 15 hours per week playing video games.

3. Scientists can take very good pictures of alligators from airplanes or helicopters. Scientists in Florida are interested in studying the relationship between the length and the weight of alligators in the waters around Florida.

   a.   Would it be easier to collect data on length or weight? Explain your thinking.

   b.   Use your answer to decide which variable you would want to put on the horizontal axis and which variable you might want to predict.

4. Scientists captured a small sample of alligators and measured both their length (in inches) and weight (in pounds). Torre used their data to create the following scatter plot and drew a line to capture the trend in the data. She and Steve then had a discussion about the way the line fit the data. What do you think they were discussing and why?

**Alligator Length (in.) and Weight (lb.)**



Data Source: http://exploringdata.net/stories.htm#alligatr
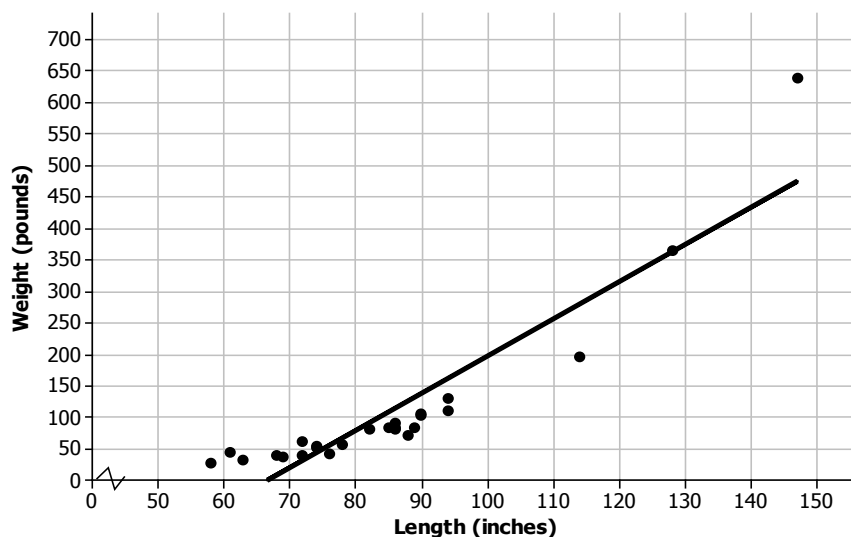
# Lesson 9:  Determining the Equation of a Line Fit to Data

## Classwork

### Example 1:  Crocodiles and Alligators

Scientists are interested in finding out how different species adapt to finding food sources.  One group studied crocodilian to find out how their bite force was related to body mass and diet.  The table below displays the information they collected on body mass (in pounds) and bite force (in pounds).

**Crocodilian Biting**

| Species | Body Mass (pounds) | Bite Force (pounds) |
|---|---|---|
| Dwarf Crocodile | 35 | 450 |
| Crocodile F | 40 | 260 |
| Alligator A | 30 | 250 |
| Caiman A | 28 | 230 |
| Caiman B | 37 | 240 |
| Caiman C | 45 | 255 |
| Crocodile A | 110 | 550 |
| Nile Crocodile | 275 | 650 |
| Crocodile B | 130 | 500 |
| Crocodile C | 135 | 600 |
| Crocodile D | 135 | 750 |
| Caiman D | 125 | 550 |
| Indian Gharial Crocodile | 225 | 400 |
| Crocodile G | 220 | 1,000 |
| American Crocodile | 270 | 900 |
| Crocodile D | 285 | 750 |
| Crocodile E | 425 | 1,650 |
| American Alligator | 300 | 1,150 |
| Alligator B | 325 | 1,200 |
| Alligator C | 365 | 1,450 |

Data Source:  PLoS One Greg Erickson biomechanics, Florida State University

As you learned in the previous lesson, it is a good idea to begin by looking at what a scatter plot tells you about the data. The scatter plot below displays the data on body mass and bite force for the crocodilian in the study.
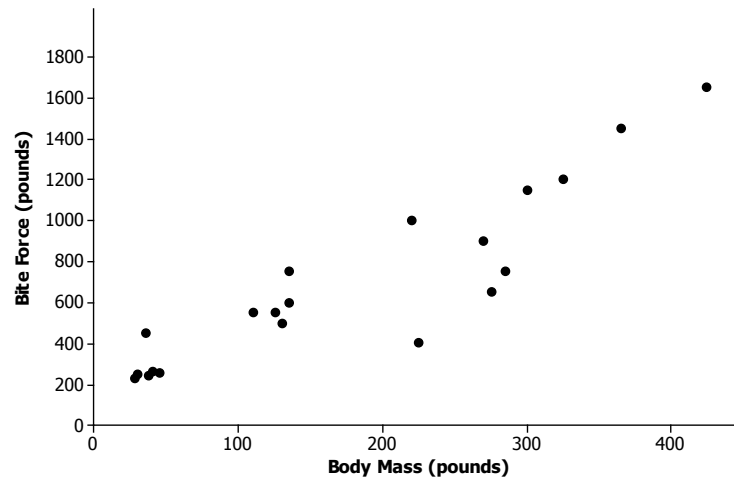


**Exercises 1–6**

1.  Describe the relationship between body mass and bite force for the crocodilian shown in the scatter plot.

2.  Draw a line to represent the trend in the data.  Comment on what you considered in drawing your line.

3.  Based on your line, predict the bite force for a crocodilian that weighs 220 pounds.  How does this prediction compare to the actual bite force of the 220-pound crocodilian in the data set?

4.  Several students decided to draw lines to represent the trend in the data. Consider the lines drawn by Sol, Patti, Marrisa, and Taylor, which are shown below.
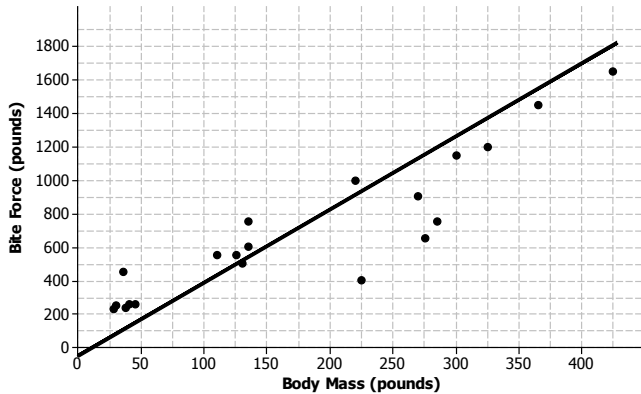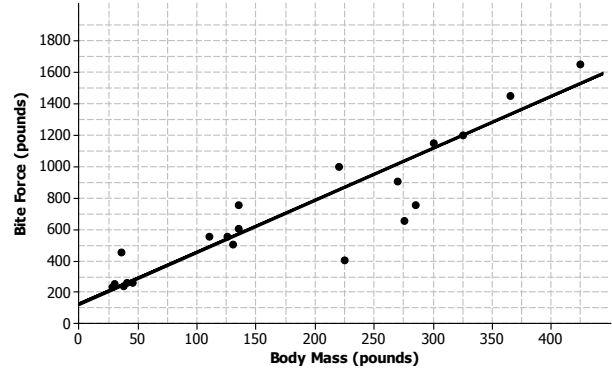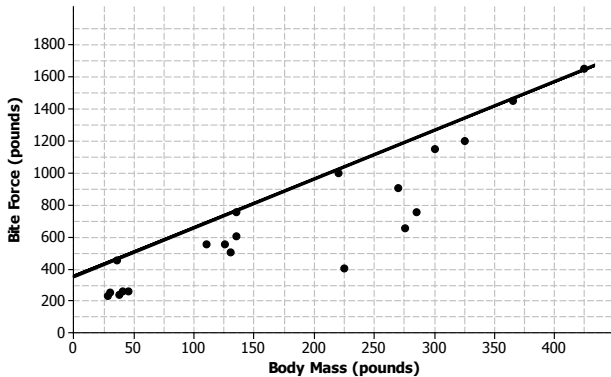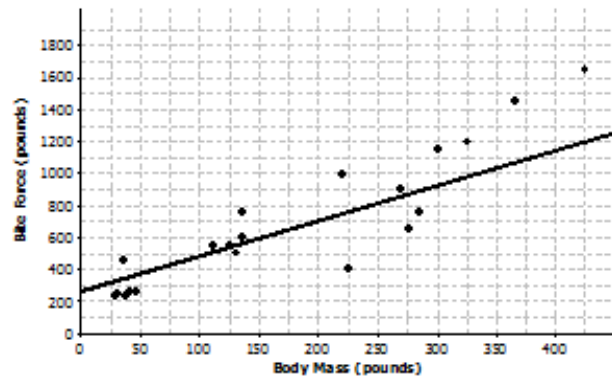


For each student, indicate whether or not you think the line would be a good line to use to make predictions. Explain your thinking.

a.  Sol's line

b.  Patti's line

c.  Marrisa's line

d.  Taylor's line

5.　What is the equation of your line?  Show the steps you used to determine your line.  Based on your equation, what is your prediction for the bite force of a crocodilian weighing 200 pounds?

6.　Patti drew vertical line segments from two points to the line in her scatter plot.  The first point she selected was for a Dwarf Crocodile.  The second point she selected was for an Indian Gharial Crocodile.
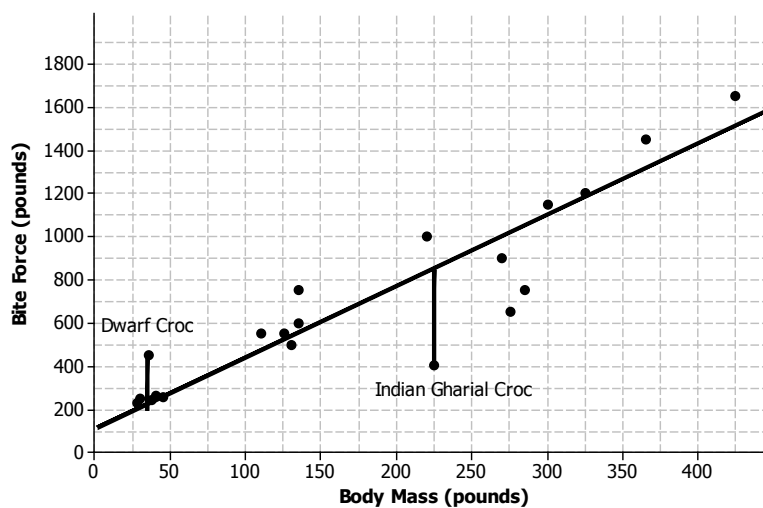


a.　Would Patti's line have resulted in a predicted bite force that was closer to the actual bite force for the Dwarf Crocodile or for the Indian Gharial Crocodile?  What aspect of the scatter plot supports your answer?

b.　Would it be preferable to describe the trend in a scatter plot using a line that makes the differences in the actual and predicted values large or small?  Explain your answer.

### Exercise 7: Used Cars

7.  The plot below shows the age (in years) and price (in dollars) of used Honda Civic cars that were advertised in a local newspaper.
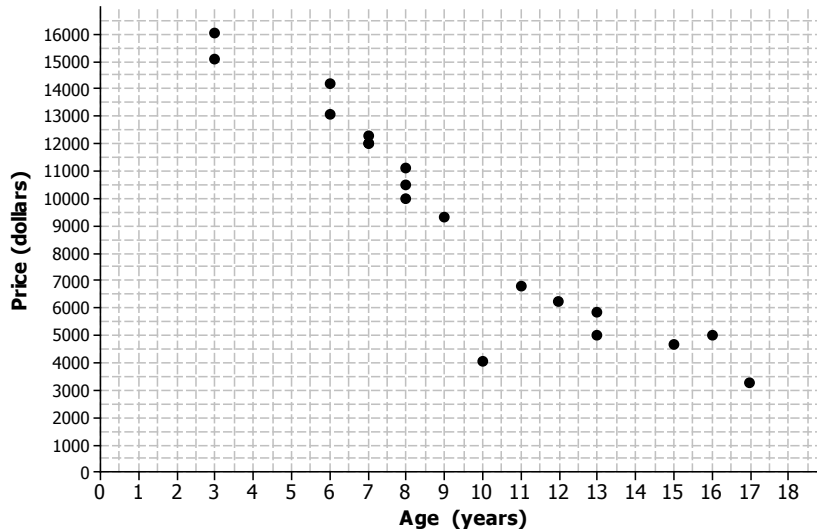


a.  Based on the scatter plot above, describe the relationship between the age and price of the used cars.



b.  Nora drew a line she thought was close to many of the points and found the equation of the line. She used the points $(13, 6000)$ and $(7, 12000)$ on her line to find the equation. Explain why those points made finding the equation easy.
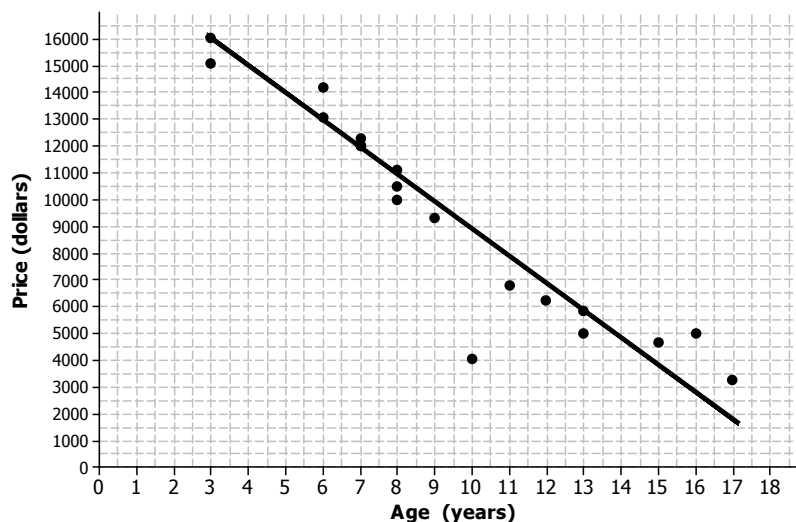
c.   Find the equation of Nora's line for predicting the price of a used car given its age.  Summarize the trend described by this equation.

d.   Based on the line, for which car in the data set would the predicted value based on the line be farthest from the actual value?  How can you tell?

e.   What does the equation predict for the cost of a 10-year-old car?  How close was the prediction using the line to the actual cost of the 10-year-old car in the data set?  Given the context of the data set, do you think the difference between the predicted price and the actual price is large or small?

f.   Is $5,000 typical of the differences between predicted prices and actual prices for the cars in this data set? Justify your answer.

<div style="border:1px solid #1a6ca0; padding:10px;">

**Lesson Summary**

- A line can be used to represent the trend in a scatter plot.
- Evaluating the equation of the line for a value of the independent variable will determine a value predicted by the line.
- A good line for prediction is one that goes through the middle of the points in a scatter plot and for which the points tend to fall close to the line.

</div>

## Problem Set

1. Monopoly is a popular board game in many countries. The scatter plot below shows the distance from "Go" to a property (in number of spaces moving from "Go" in a clockwise direction) and the price of the properties on the Monopoly board. The equation of the line is $P = 8x + 40$, where $P$ represents the price (in Monopoly dollars) and $x$ represents the distance (in number of spaces).

| Distance from "Go" (number of spaces) | Price of Property (Monopoly dollars) | Distance from "Go" (number of spaces) | Price of Property (Monopoly dollars) |
|---|---|---|---|
| 1 | 60 | 21 | 220 |
| 3 | 60 | 23 | 220 |
| 5 | 200 | 24 | 240 |
| 6 | 100 | 25 | 200 |
| 8 | 100 | 26 | 260 |
| 9 | 120 | 27 | 260 |
| 11 | 140 | 28 | 150 |
| 12 | 150 | 29 | 280 |
| 13 | 140 | 31 | 300 |
| 14 | 160 | 32 | 300 |
| 15 | 200 | 34 | 320 |
| 16 | 180 | 35 | 200 |
| 18 | 180 | 37 | 350 |
| 19 | 200 | 39 | 400 |

**Price of Property vs. Distance from "Go" in Monopoly**
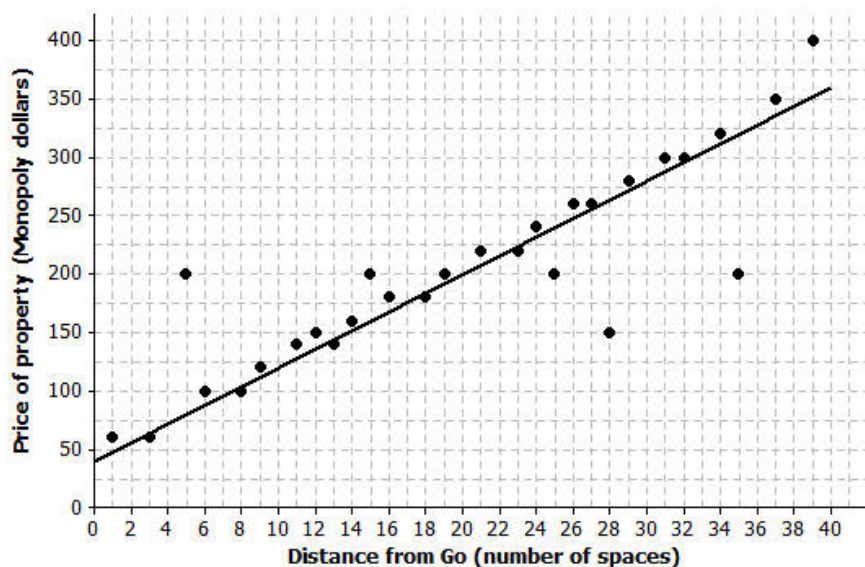


a.  Use the equation to find the difference (observed value − predicted value) for the most expensive property and for the property that is 35 spaces from "Go."

b.  Five of the points seem to lie in a horizontal line. What do these points have in common? What is the equation of the line containing those five points?

c.  Four of the five points described in part (b) are the railroads. If you were fitting a line to predict price with distance from "Go," would you use those four points? Why or why not?

2.  The table below gives the coordinates of the five points shown in the scatter plots that follow. The scatter plots show two different lines.

| Data Point | Independent Variable | Response Variable |
|---|---|---|
| A | 20 | 27 |
| B | 22 | 21 |
| C | 25 | 24 |
| D | 31 | 18 |
| E | 40 | 12 |

**Line 1**

**Line 2**



$y = -0.9x + 45$

$y = -0.7x + 40$
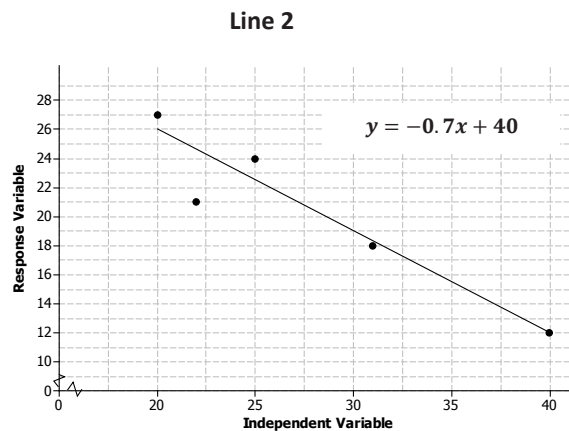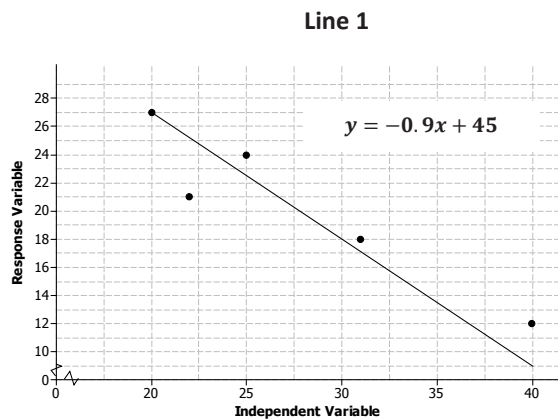
a.  Find the predicted response values for each of the two lines.

| Independent | Observed Response | Response Predicted by Line 1 | Response Predicted by Line 2 |
|---|---|---|---|
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

b.  For which data points is the prediction based on Line 1 closer to the actual value than the prediction based on Line 2?

c.  Which line (Line 1 or Line 2) would you select as a better fit?

3. The scatter plots below show different lines that students used to model the relationship between body mass (in pounds) and bite force (in pounds) for crocodilian.

   a. Match each graph to one of the equations below and explain your reasoning. Let $B$ represent bite force (in pounds) and $W$ represent body mass (in pounds).

   | Equation 1 | Equation 2 | Equation 3 |
   |:---:|:---:|:---:|
   | $B = 3.28W + 126$ | $B = 3.04W + 351$ | $B = 2.16W + 267$ |

   | Equation: | |
   |---|---|
   | | 

   **Line 1** |

   | Equation: | |
   |---|---|
   | | 
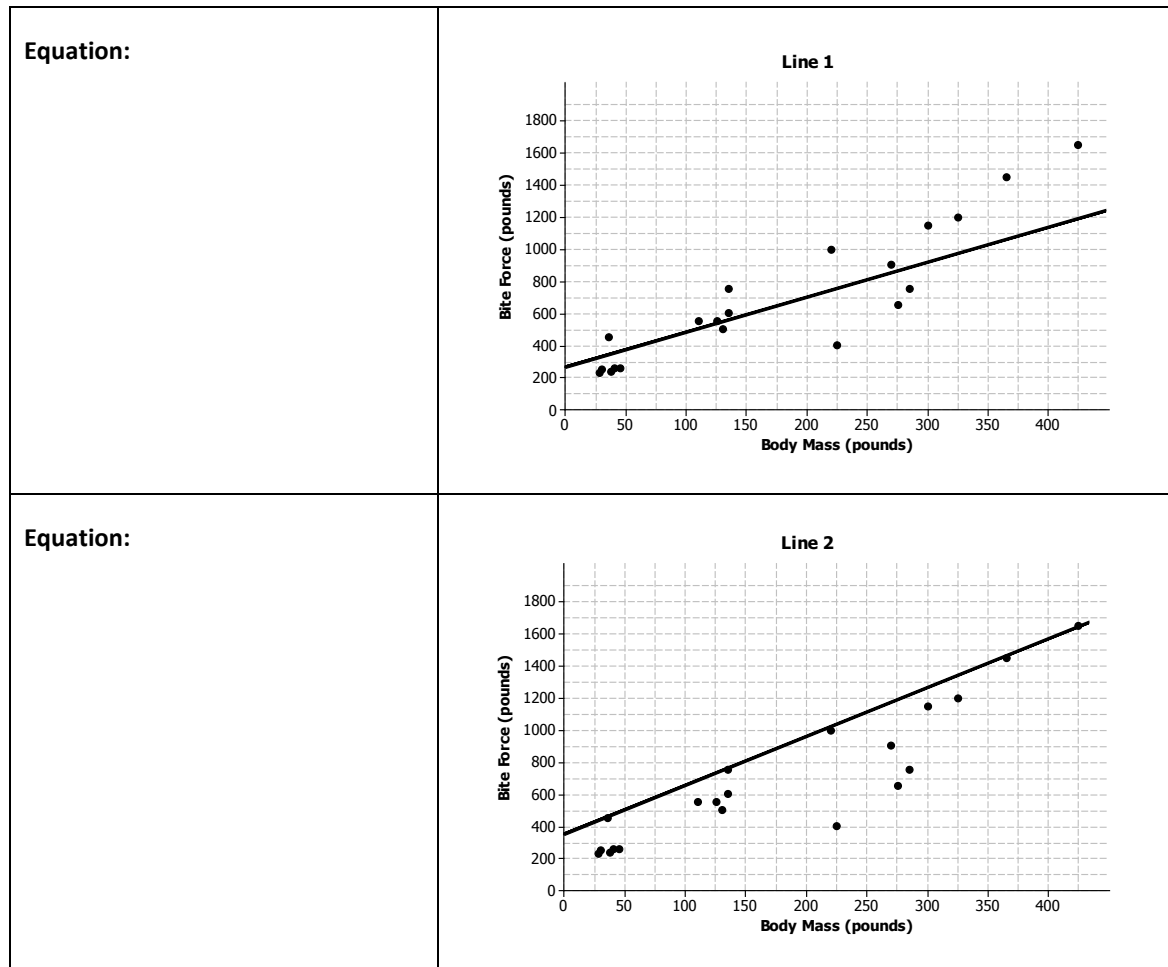
   **Line 2** |

| Equation: |  |
| --- | --- |
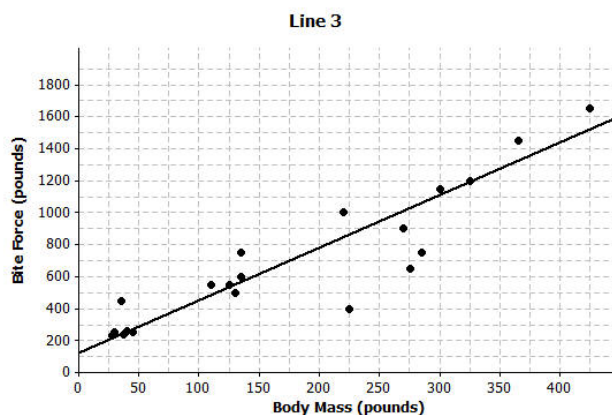
b.   Which of the lines would best fit the trend in the data?  Explain your thinking.

4.   Comment on the following statements:

a.   A line modeling a trend in a scatter plot always goes through the origin.

b.   If the response variable increases as the independent variable decreases, the slope of a line modeling the trend will be negative.

# Lesson 10:  Linear Models

## Classwork

In previous lessons, you used data that follow a linear trend either in the positive direction or the negative direction and informally fitted a line through the data.  You determined the equation of an informal fitted line and used it to make predictions.

In this lesson, you will use a function to model a linear relationship between two numerical variables and interpret the slope and intercept of the linear model in the context of the data.  Recall that a function is a rule that relates a dependent variable to an independent variable.

In statistics, a dependent variable is also called a response variable or a predicted variable.  An independent variable is also called an explanatory variable or a predictor variable.

### Example 1

Predicting the value of a numerical dependent (response) variable based on the value of a given numerical independent variable has many applications in statistics.  The first step in the process is to identify the dependent (predicted) variable and the independent (predictor) variable.

There may be several independent variables that might be used to predict a given dependent variable.  For example, suppose you want to predict how well you are going to do on an upcoming statistics quiz.  One possible independent variable is how much time you spent studying for the quiz.  What are some other possible numerical independent variables that could relate to how well you are going to do on the quiz?

### Exercises 1–2

1.  For each of the following dependent (response) variables, identify two possible numerical independent (explanatory) variables that might be used to predict the value of the dependent variable.

| Response Variable | Possible Explanatory Variables |
|---|---|
| Height of a son | |
| Number of points scored in a game by a basketball player | |
| Number of hamburgers to make for a family picnic | |
| Time it takes a person to run a mile | |
| Amount of money won by a contestant on Jeopardy! (television game show) | |

| Response Variable | Possible Explanatory Variables |
|---|---|
| Fuel efficiency (in miles per gallon) for a car | |
| Number of honey bees in a beehive at a particular time | |
| Number of blooms on a dahlia plant | |
| Number of forest fires in a state during a particular year | |

2.  Now, reverse your thinking. For each of the following numerical independent variables, write a possible numerical dependent variable.

| Dependent Variable | Possible Independent Variables |
|---|---|
| | Age of a student |
| | Height of a golfer |
| | Amount of a pain-reliever taken |
| | Number of years of education |
| | Amount of fertilizer used on a garden |
| | Size of a diamond in a ring |
| | Total salary for all of a team's players |

## Example 2

A cell-phone company offers the following basic cell-phone plan to its customers:  A customer pays a monthly fee of $40.00.  In addition, the customer pays $0.15 per text message sent from the cell phone.  There is no limit to the number of text messages per month that could be sent, and there is no charge for receiving text messages.

**Exercises 3–11**

3.  Determine the following:

    a.  Justin never sends a text message.  What would be his total monthly cost?

    b.  During a typical month, Abbey sends 25 text messages.  What is her total cost for a typical month?

    c.  Robert sends at least 250 text messages a month.  What would be an estimate of the least his total monthly cost is likely to be?

4.  Use descriptive words to write a linear model describing the relationship between the number of text messages sent and the total monthly cost.

5.  Is the relationship between the number of text messages sent and the total monthly cost linear?  Explain your answer.

6.  Let $x$ represent the independent variable and $y$ represent the dependent variable.  Use the variables $x$ and $y$ to write the function representing the relationship you indicated in Exercise 4.

7.  Explain what $0.15 represents in this relationship.

8.  Explain what $40.00 represents in this relationship.

9.  Sketch a graph of this relationship on the following coordinate grid.  Clearly label the axes and include units in the labels.
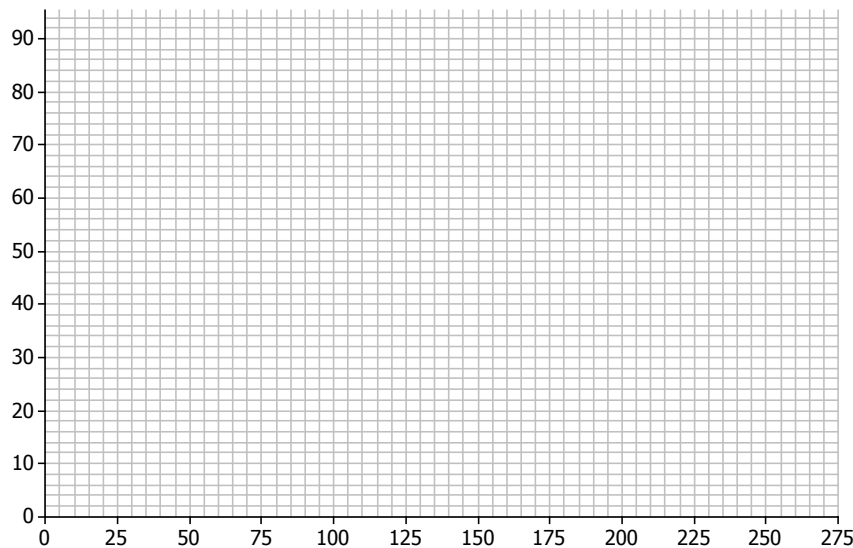
10. LaMoyne needs four more pieces of lumber for his scout project.  The pieces can be cut from one large piece of lumber according to the following pattern.



The lumberyard will make the cuts for LaMoyne at a fixed cost of $2.25 plus an additional cost of 25 cents per cut.  One cut is free.

a.  What is the functional relationship between the total cost of cutting a piece of lumber and the number of cuts required?  What is the equation of this function?  Be sure to define the variables in the context of this problem.

b.  Use the equation to determine LaMoyne's total cost for cutting.

c.  In the context of this problem, interpret the slope of the equation in words.

d.  Interpret the $y$-intercept of your equation in words in the context of this problem.  Does interpreting the intercept make sense in this problem?  Explain.

11. Omar and Olivia were curious about the size of coins.  They measured the diameter and circumference of several coins and found the following data.

| US Coin | Diameter (mm) | Circumference (mm) |
|---------|---------------|--------------------|
| Penny | 19.0 | 59.7 |
| Nickel | 21.2 | 66.6 |
| Dime | 17.9 | 56.2 |
| Quarter | 24.3 | 76.3 |
| Half Dollar | 30.6 | 96.1 |

a. Wondering if there was any relationship between diameter and circumference, they thought about drawing a picture.  Draw a scatter plot that displays circumference in terms of diameter.

b. Do you think that circumference and diameter are related?  Explain.

c.   Find the equation of the function relating circumference to the diameter of a coin.

d.   The value of the slope is approximately equal to the value of $\pi$.  Explain why this makes sense.

e.   What is the value of the $y$-intercept?  Explain why this makes sense.

> **Lesson Summary**
>
> - A linear functional relationship between a dependent and independent numerical variable has the form $y = mx + b$ or $y = a + bx$.
> - In statistics, a dependent variable is one that is predicted and an independent variable is the one that is used to make the prediction.
> - The graph of a linear function describing the relationship between two variables is a line.

## Problem Set

1. The Mathematics Club at your school is having a meeting. The advisor decides to bring bagels and his award-winning strawberry cream cheese. To determine his cost, from past experience he figures 1.5 bagels per student. A bagel costs 65 cents, and the special cream cheese costs $3.85 and will be able to serve all of the anticipated students attending the meeting

   a. Find an equation that relates his total cost to the number of students he thinks will attend the meeting.

   b. In the context of the problem, interpret the slope of the equation in words.

   c. In the context of the problem, interpret the $y$-intercept of the equation in words. Does interpreting the intercept make sense? Explain.

2. John, Dawn, and Ron get together to exercise (walk/jog) for 45 minutes. John has arthritic knees but manages to walk $1\frac{1}{2}$ miles. Dawn walks $2\frac{1}{4}$ miles, while Ron manages to jog 6 miles.

   a. Draw an appropriate graph and connect the points to show that there is a linear relationship between the distance that each traveled based on how fast each traveled (speed). Note that the speed for a person who travels 3 miles in 45 minutes, or $\frac{3}{4}$ hours, is $3 \div \frac{3}{4} = 4$ miles per hour.

   b. Find an equation that expresses distance in terms of speed (how fast one goes).

   c. In the context of the problem, interpret the slope of the equation in words.

   d. In the context of the problem, interpret the $y$-intercept of the equation in words. Does interpreting the intercept make sense? Explain.

3.  Simple interest is money that is paid on a loan.  Simple interest is calculated by taking the amount of the loan and multiplying it by the rate of interest per year and the number of years the loan is outstanding.  For college, Jodie's older brother has taken out a student loan for $4,500 at an interest rate of 5.6%, or 0.056.  When he graduates in four years, he will have to pay back the loan amount plus interest for four years.  Jodie is curious as to how much her brother will have to pay.

    a.  Jodie claims that his brother will have to pay her a total of $5,508.  Do you agree?  Explain.  As an example, 8% simple interest on $1,200 for one year is $(0.08)(\$1200) = \$96$.  The interest for two years would be $2 \times \$96$, or $192.

    b.  Write an equation for the total cost to repay a loan of $P$ if the rate of interest for a year is $r$ (expressed as a decimal) for a time span of $t$ years.

    c.  If $P$ and $r$ are known, is the equation a linear equation?

    d.  In the context of the problem, interpret the slope of the equation in words.

    e.  In the context of the problem, interpret the intercept of the equation in words.  Does interpreting the intercept make sense?  Explain.

# Lesson 11:  Using Linear Models in a Data Context

## Classwork

### Exercises

1.  Old Faithful is a geyser in Yellowstone National Park.  The following table offers some rough estimates of the length of an eruption (in minutes) and the amount of water (in gallons) in that eruption.

| Length (min.) | 1.5 | 2 | 3 | 4.5 |
|---|---|---|---|---|
| Amount of Water (gal.) | 3,700 | 4,100 | 6,450 | 8,400 |

   a.   Chang wants to predict the amount of water in an eruption based on the length of the eruption.  What should he use as the dependent variable?  Why?

   b.   Which of the following two scatter plots should Chang use to build his prediction model?  Explain.
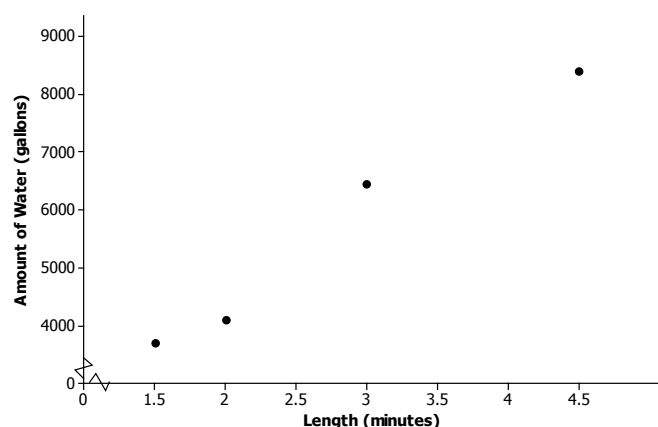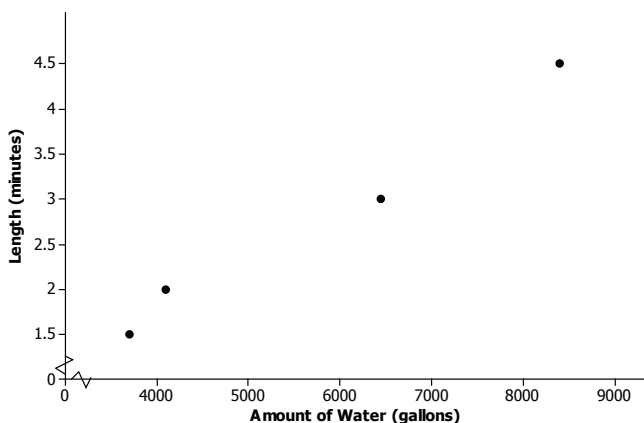
c.  Suppose that Chang believes the variables to be linearly related.  Use the *first* and *last* data points in the table to create a linear prediction model.

d.  A friend of Chang's told him that Old Faithful produces about 3,000 gallons of water for every minute that it erupts.  Does the linear model from part (c) support what Chang's friend said?  Explain.

e.  Using the linear model from part (c), does it make sense to interpret the $y$-intercept in the context of this problem?  Explain.

2. The following table gives the times of the gold, silver, and bronze medal winners for the men's 100 meter race (in seconds) for the past 10 Olympic Games.

| Year | 2012 | 2008 | 2004 | 2000 | 1996 | 1992 | 1988 | 1984 | 1980 | 1976 |
|------|------|------|------|------|------|------|------|------|------|------|
| Gold | 9.63 | 9.69 | 9.85 | 9.87 | 9.84 | 9.96 | 9.92 | 9.99 | 10.25 | 10.06 |
| Silver | 9.75 | 9.89 | 9.86 | 9.99 | 9.89 | 10.02 | 9.97 | 10.19 | 10.25 | 10.07 |
| Bronze | 9.79 | 9.91 | 9.87 | 10.04 | 9.90 | 10.04 | 9.99 | 10.22 | 10.39 | 10.14 |
| Mean Time | 9.72 | 9.83 | 9.86 | 9.97 | 9.88 | 10.01 | 9.96 | 10.13 | 10.30 | 10.09 |

a. If you wanted to describe how mean times change over the years, which variable would you use as the independent variable, and which would you use as the dependent variable?

b. Draw a scatter plot to determine if the relationship between mean time and year appears to be linear. Comment on any trend or pattern that you see in the scatter plot.

c. One reasonable line goes through the 1992 and 2004 data. Find the equation of that line.

d. Before he saw these data, Chang guessed that the mean time of the three Olympic medal winners decreased by about 0.05 seconds from one Olympic Games to the next. Does the prediction model you found in part (c) support his guess? Explain.

e. If the trend continues, what mean race time would you predict for the gold, silver, and bronze medal winners in the 2016 Olympic Games? Explain how you got this prediction.

f. The data point $(1980, 10.3)$ appears to have an unusually high value for the mean time $(10.3)$. Using your library or the Internet, see if you can find a possible explanation for why that might have happened.

## Lesson Summary

In the real world, it is rare that two numerical variables are exactly linearly related. If the data are roughly linearly related, then a line can be drawn through the data. This line can then be used to make predictions and to answer questions. For now, the line is informally drawn, but in later grades you will see more formal methods for determining a best-fitting line.

## Problem Set

1.  From the United States Bureau of Census website, the population sizes (in millions of people) in the United States for census years 1790–2010 are as follows.

| Year | 1790 | 1800 | 1810 | 1820 | 1830 | 1840 | 1850 | 1860 | 1870 | 1880 | 1890 |
|------|------|------|------|------|------|------|------|------|------|------|------|
| Population Size | 3.9 | 5.3 | 7.2 | 9.6 | 12.9 | 17.1 | 23.2 | 31.4 | 38.6 | 50.2 | 63.0 |

| Year | 1900 | 1910 | 1920 | 1930 | 1940 | 1950 | 1960 | 1970 | 1980 | 1990 | 2000 | 2010 |
|------|------|------|------|------|------|------|------|------|------|------|------|------|
| Population Size | 76.2 | 92.2 | 106.0 | 123.2 | 132.2 | 151.3 | 179.3 | 203.3 | 226.5 | 248.7 | 281.4 | 308.7 |

  a.  If you wanted to be able to predict population size in a given year, which variable would be the independent variable and which would be the dependent variable?

  b.  Draw a scatter plot. Does the relationship between year and population size appear to be linear?

  c.  Consider the data only from 1950 to 2010. Does the relationship between year and population size for these years appear to be linear?

  d.  One line that could be used to model the relationship between year and population size for the data from 1950 to 2010 is $y = -4875.021 + 2.578x$. Suppose that a sociologist believes that there will be negative consequences if population size in the United States increases by more than $2\frac{3}{4}$ million people annually. Should she be concerned? Explain your reasoning.

  e.  Assuming that the linear pattern continues, use the line given in part (d) to predict the size of the population in the United States in the next census.

2.  In search of a topic for his science class project, Bill saw an interesting YouTube video in which dropping mint candies into bottles of a soda pop caused the pop to spurt immediately from the bottle. He wondered if the height of the spurt was linearly related to the number of mint candies that were used. He collected data using 1, 3, 5, and 10 mint candies. Then he used two-liter bottles of a diet soda and measured the height of the spurt in centimeters. He tried each quantity of mint candies three times. His data are in the following table.

| Number of Mint Candies | 1 | 1 | 1 | 3 | 3 | 3 | 5 | 5 | 5 | 10 | 10 | 10 |
|------|---|---|---|---|---|---|---|---|---|----|----|----|
| Height of Spurt (cm) | 40 | 35 | 30 | 110 | 105 | 90 | 170 | 160 | 180 | 400 | 390 | 420 |

  a.  Identify which variable is the independent variable and which is the dependent variable.

  b.  Draw a scatter plot that could be used to determine whether the relationship between height of spurt and number of mint candies appears to be linear.

c.   Bill sees a slight curvature in the scatter plot, but he thinks that the relationship between the number of mint candies and the height of the spurt appears close enough to being linear, and he proceeds to draw a line.  His eyeballed line goes through the mean of the three heights for three mint candies and the mean of the three heights for 10 candies.  Bill calculates the equation of his eyeballed line to be

$$y = -27.617 + (43.095)x,$$

where the height of the spurt ($y$) in centimeters is based on the number of mint candies ($x$).  Do you agree with this calculation?  He rounded all of his calculations to three decimal places.  Show your work.

d.   In the context of this problem, interpret in words the slope and intercept for Bill's line.  Does interpreting the intercept make sense in this context?  Explain.

e.   If the linear trend continues for greater numbers of mint candies, what would you predict the height of the spurt will be if 15 mint candies are used?

# Lesson 12:  Nonlinear Models in a Data Context

## Classwork

### Example 1:  Growing Dahlias

A group of students wanted to determine whether or not compost is beneficial in plant growth.  The students used the dahlia flower to study the effect of composting.  They planted eight dahlias in a bed with no compost and another eight plants in a bed with compost.  They measured the height of each plant over a 9-week period.  They found the median growth height for each group of eight plants.  The table below shows the results of the experiment for the dahlias grown in non-compost beds.

| Week | Median Height in Non-Compost Bed (inches) |
|------|-------------------------------------------|
| 1    | 9.00                                      |
| 2    | 12.75                                     |
| 3    | 16.25                                     |
| 4    | 19.50                                     |
| 5    | 23.00                                     |
| 6    | 26.75                                     |
| 7    | 30.00                                     |
| 8    | 33.75                                     |
| 9    | 37.25                                     |

### Exercises 1–15

1.  On the grid below, construct a scatter plot of non-compost data.
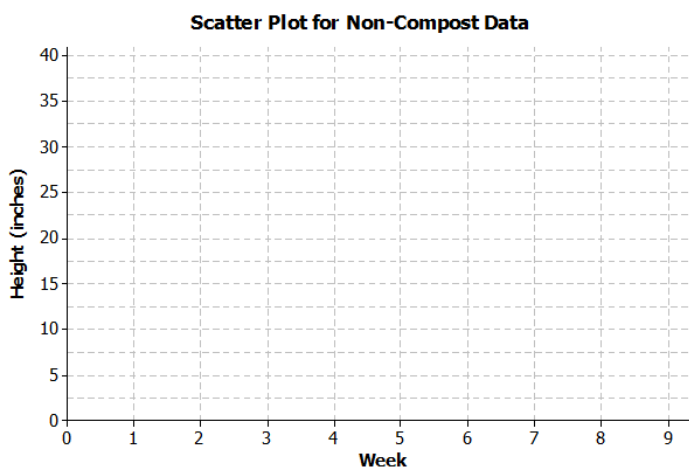


2.  Draw a line that you think fits the data reasonably well.

3. Find the rate of change of your line. Interpret the rate of change in terms of growth (in height) over time.

4. Describe the growth (change in height) from week to week by subtracting the previous week's height from the current height. Record the weekly growth in the third column in the table below. The median growth for the dahlias from Week 1 to Week 2 was 3.75 inches (i.e., $12.75 - 9.00 = 3.75$).

| Week | Median Height in Non-Compost Bed (inches) | Weekly Growth (inches) |
|---|---|---|
| 1 | 9.00 | — |
| 2 | 12.75 | 3.75 |
| 3 | 16.25 | |
| 4 | 19.50 | |
| 5 | 23.00 | |
| 6 | 26.75 | |
| 7 | 30.00 | |
| 8 | 33.75 | |
| 9 | 37.25 | |

5. As the number of weeks increases, describe how the weekly growth is changing.

6. How does the growth each week compare to the slope of the line that you drew?

7. Estimate the median height of the dahlias at $8\frac{1}{2}$ weeks. Explain how you made your estimate.

The table below shows the results of the experiment for the dahlias grown in compost beds.

| Week | Median Height in Compost Bed (inches) |
|------|---------------------------------------|
| 1 | 10.00 |
| 2 | 13.50 |
| 3 | 17.75 |
| 4 | 21.50 |
| 5 | 30.50 |
| 6 | 40.50 |
| 7 | 65.00 |
| 8 | 80.50 |
| 9 | 91.50 |

8.  Construct a scatter plot of height versus week on the grid below.

**Scatter Plot for Compost Data**
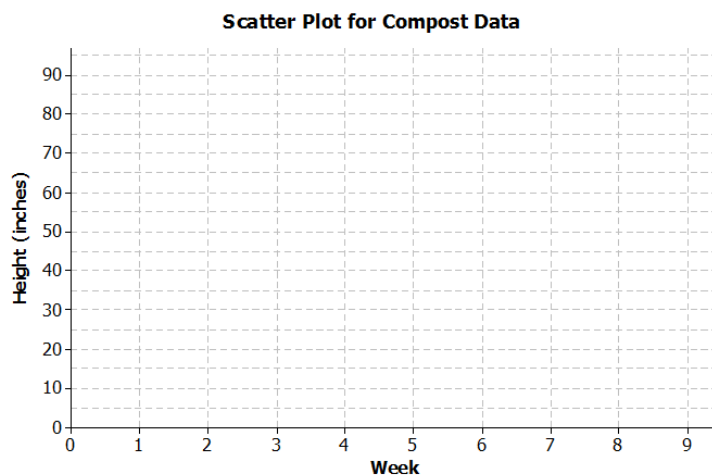
9.  Do the data appear to form a linear pattern?

10. Describe the weekly growth from week to week by subtracting the height from the previous week from the current height. Record the weekly growth in the third column in the table below. The median weekly growth for the dahlias from Week 1 to Week 2 is 3.5 inches. (i.e., $13.5 - 10 = 3.5$).

| Week | Compost Height (inches) | Weekly Growth (inches) |
|---|---|---|
| 1 | 10.00 | — |
| 2 | 13.50 | 3.50 |
| 3 | 17.75 | |
| 4 | 21.50 | |
| 5 | 30.50 | |
| 6 | 40.50 | |
| 7 | 65.00 | |
| 8 | 80.50 | |
| 9 | 91.50 | |

11. As the number of weeks increases, describe how the growth changes.

12. Sketch a curve through the data. When sketching a curve do not connect the ordered pairs, but draw a smooth curve that you think reasonably describes the data.

13. Use the curve to estimate the median height of the dahlias at $8\frac{1}{2}$ weeks. Explain how you made your estimate.

14. How does the weekly growth of the dahlias in the compost beds compare to the weekly growth of the dahlias in the non-compost beds?

15. When there is a car accident how do the investigators determine the speed of the cars involved?  One way is to measure the skid marks left by the car and use this length to estimate the speed.

    The table below shows data collected from an experiment with a test car.  The first column is the length of the skid mark (in feet) and the second column is the speed of the car (in miles per hour).

    | Skid-Mark Length (ft.) | Speed (mph) |
    |---|---|
    | 5 | 10 |
    | 17 | 20 |
    | 65 | 40 |
    | 105 | 50 |
    | 205 | 70 |
    | 265 | 80 |

    a.  Construct a scatter plot of speed versus skid-mark length on the grid below.
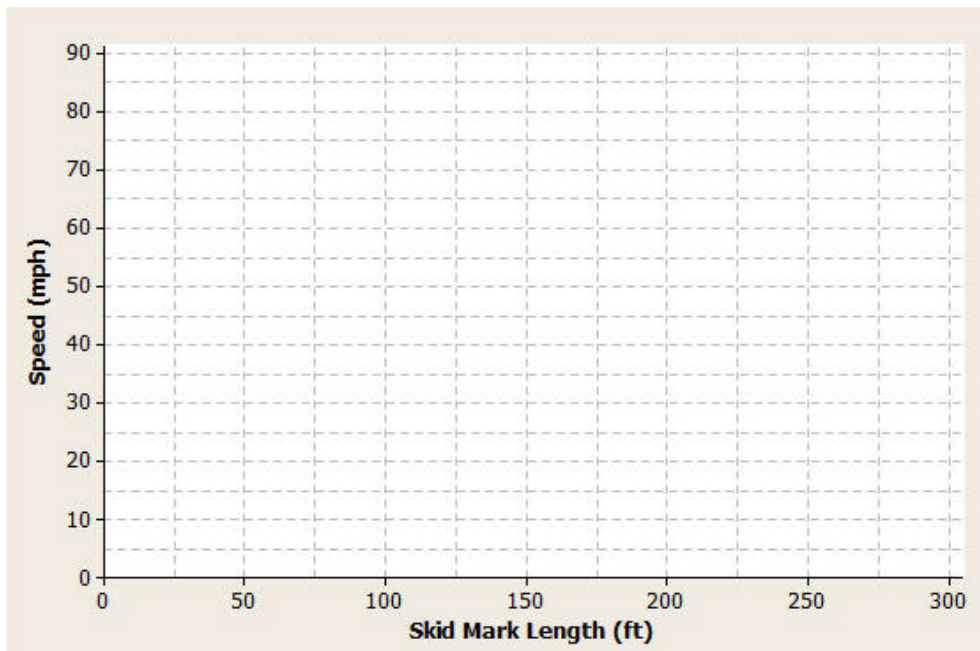


    b.  The relationship between speed and skid-mark length can be described by a curve. Sketch a curve through the data that best represents the relationship between skid-mark length and speed of the car.  Remember to draw a smooth curve that does not just connect the ordered pairs.

c.   If the car left a skid mark of 60 ft., what is an estimate for the speed of the car?  Explain how you determined the estimate.

d.   A car left a skid mark of 150 ft.  Use the curve you sketched to estimate the speed at which the car was traveling.

e.   If a car leaves a skid mark that is twice as long as another skid mark, was the car going twice as fast?  Explain.

**Lesson Summary**

When data follow a linear pattern, the rate of change is a constant. When data follow a nonlinear pattern, the rate of change is not constant.

## Problem Set

1.  Once the brakes of the car have been applied, the car does not stop immediately. The distance that the car travels after the brakes have been applied is called the braking distance. The table below shows braking distance (how far the car travels once the brakes have been applied) and the speed of the car.

| Speed (mph) | Distance Until Car Stops (ft.) |
|---|---|
| 10 | 5 |
| 20 | 17 |
| 30 | 37 |
| 40 | 65 |
| 50 | 105 |
| 60 | 150 |
| 70 | 205 |
| 80 | 265 |

   a.  Construct a scatterplot of distance versus speed on the grid below.
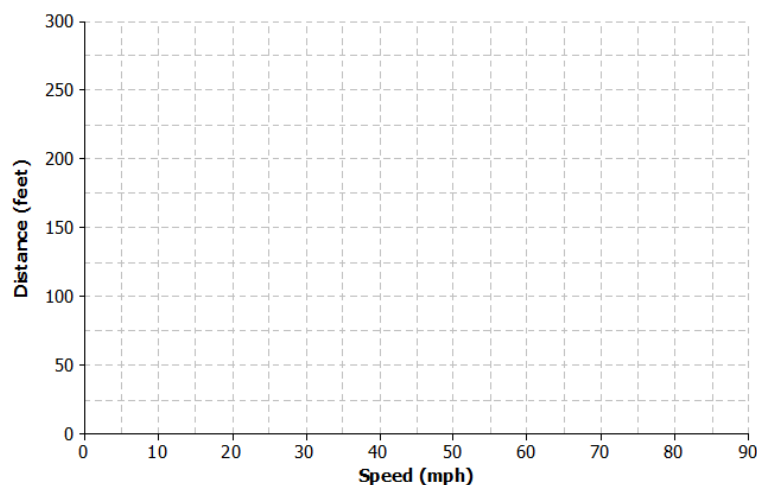
b.   Find the amount of additional distance a car would travel after braking for each speed increase of 10 mph. Record your answers in the table below.

| Speed (mph) | Distance Until Car Stops (ft.) | Amount of Distance Increase |
|---|---|---|
| 10 | 5 | — |
| 20 | 17 | |
| 30 | 37 | |
| 40 | 65 | |
| 50 | 105 | |
| 60 | 150 | |
| 70 | 205 | |
| 80 | 265 | |

c.   Based on the table, do you think the data follow a linear pattern?  Explain your answer.

d.   Describe how the distance it takes a car to stop changes as the speed of the car increases.

e.   Sketch a smooth curve that you think describes the relationship between braking distance and speed.

f.   Estimate braking distance for a car traveling at 52 mph.  Estimate braking distance for a car traveling at 75 mph.  Explain how you made your estimates.

2.   The scatter plot below shows the relationship between cost (in dollars) and radius length (in meters) of fertilizing different sized circular fields.  The curve shown was drawn to describe the relationship between cost and radius.
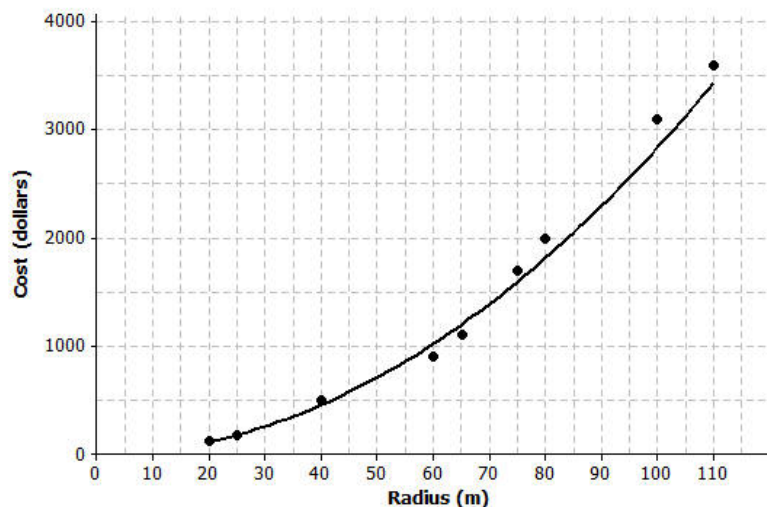


a.   Is the curve a good fit for the data?  Explain.

b.   Use the curve to estimate the cost for fertilizing a circular field of radius 30 m.  Explain how you made your estimate.

c.   Estimate the radius of the field if the fertilizing cost were $2,500.  Explain how you made your estimate.

3. A dolphin is fitted with a GPS system that monitors its position in relationship to a research ship. The table below contains the time (in seconds) after the dolphin is released from the ship and the distance (in feet) the dolphin is from the research ship.

| Time (sec.) | Distance from Ship (ft.) | Increase in Distance from the Ship |
|---|---|---|
| 0 | 0 | — |
| 50 | 85 | |
| 100 | 190 | |
| 150 | 398 | |
| 200 | 577 | |
| 250 | 853 | |
| 300 | 1,122 | |

a. Construct a scatter plot of distance versus time on the grid below.
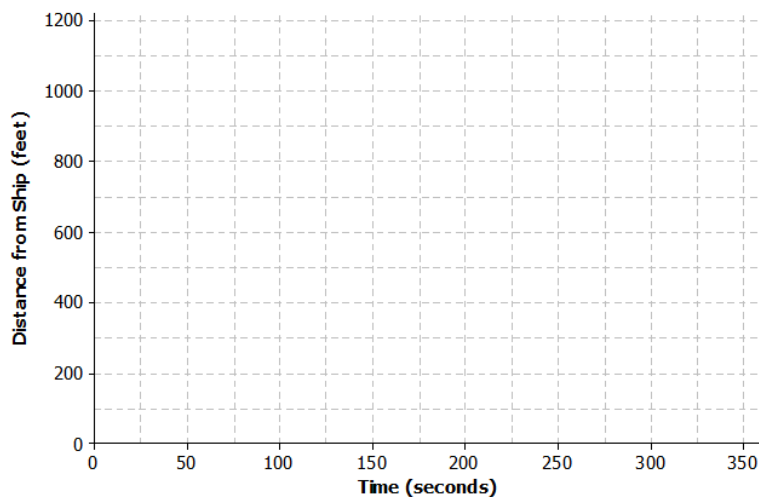


b. Find the additional distance the dolphin traveled for each increase of 50 seconds. Record your answers in the table above.

c. Based on the table, do you think that the data follow a linear pattern? Explain your answer.

d. Describe how the distance that the dolphin is from the ship changes as the time increases.

e. Sketch a smooth curve that you think fits the data reasonably well.

f. Estimate how far the dolphin will be from the ship after 180 seconds? Explain how you made your estimate.

# Lesson 13: Summarizing Bivariate Categorical Data in a Two-Way Table

## Classwork

### Exercises 1–8

On an upcoming field day at school, the principal wants to provide ice cream during lunch. She will offer three flavors: chocolate, strawberry, and vanilla. She selected your class to complete a survey to help her determine how much of each flavor to buy.

1.  Answer the following question. Wait for your teacher to count how many students selected each flavor. Then, record the class totals for each flavor in the chart below.

    "Which of the following three ice cream flavors is your favorite: chocolate, strawberry, or vanilla?"

| Ice Cream Flavor | Chocolate | Strawberry | Vanilla | Total |
|---|---|---|---|---|
| **Number of Students** | | | | |

2.  Which ice cream flavor do most students prefer?

3.  Which ice cream flavor do the fewest students prefer?

4.  What percentage of students preferred each flavor? Round to the nearest tenth of a percent.

5.  Do the numbers in the chart above summarize data on a categorical variable or a numerical variable?

6.  Do the students in your class represent a random sample of all students in your school?  Why or why not?  Discuss this with your neighbor.

7.  Is your class representative of all the other classes at your school?  Why or why not?  Discuss this with your neighbor.

8.  Do you think the principal will get an accurate estimate of the proportion of students that prefer each ice cream flavor for the whole school using only your class?  Why or why not?  Discuss this with your neighbor.

<div style="background-color:#cde">

**Example 1**

</div>

Students in a different class were asked the same question about their favorite ice cream flavor.  The table below shows the ice cream flavors and the number of students who chose each flavor for that particular class.  This table is called a one-way frequency table because it shows the counts of a univariate categorical variable.

This is the univariate categorical variable.    ⟶

These are the counts for each category.    ⟶

| Ice Cream Flavor | Chocolate | Strawberry | Vanilla | Total |
|---|---|---|---|---|
| **Number of Students** | 11 | 4 | 10 | 25 |

We compute the relative frequency for each ice cream flavor by dividing the count by the total number of observations.

$$\text{relative frequency} = \frac{\text{count for a category}}{\text{total number of observations}}$$

Since 11 out of 25 students answered chocolate, the relative frequency would be $\frac{11}{25} = 0.44$.  This relative frequency shows that $44\%$ of the class prefers chocolate ice cream.  In other words, the relative frequency is the proportional value that each category is of the whole.

## Exercises 9–10

Use the table for the preferred ice cream flavors from the class in Example 1 to answer the following questions.

9. What is the relative frequency for the category strawberry?

10. Write a sentence interpreting the relative frequency value in the context of strawberry ice cream preference.

## Example 2

The principal also wondered if boys and girls have different favorite ice cream flavors. She decided to redo the survey by taking a random sample of students from the school and recording both their favorite ice cream flavor and their gender. She asked the following two questions:

- "Which of the following ice cream flavors is your favorite: chocolate, strawberry, or vanilla?"
- "What is your gender: male or female?"

The results of the survey are as follows:

- Of the 30 students who prefer chocolate ice cream, 22 are males.
- Of the 25 students who prefer strawberry ice cream, 15 are females.
- Of the 27 students who prefer vanilla ice cream, 13 are males.

The values of two variables, which were ice cream flavor and gender, were recorded in this survey. Since both of the variables are categorical, the data are bivariate categorical data.

## Exercises 11–17

11. Can we display these data in a one-way frequency table? Why or why not?

12. Summarize the results of the second survey of favorite ice cream flavors in the following table:

|  |  | Favorite Ice Cream Flavor | | | |
|---|---|---|---|---|---|
|  |  | Chocolate | Strawberry | Vanilla | Total |
| Gender | Male |  |  |  |  |
|  | Female |  |  |  |  |
|  | Total |  |  |  |  |

13. Calculate the relative frequencies for the table above and write them in the table.

|  |  | Favorite Ice Cream Flavor | | | |
|---|---|---|---|---|---|
|  |  | Chocolate | Strawberry | Vanilla | Total |
| Gender | Male |  |  |  |  |
|  | Female |  |  |  |  |
|  | Total |  |  |  |  |

Use the relative frequency values in the table to answer the following questions:

14. What is the proportion of the students that prefer chocolate ice cream?

15. What is the proportion of students that are female and prefer vanilla ice cream?

16. Write a sentence explaining the meaning of the approximate relative frequency 0.55.

17. Write a sentence explaining the meaning of the approximate relative frequency 0.10.

## Example 3

In the previous exercises, you used the total number of students to calculate relative frequencies. These relative frequencies were the proportion of the whole group who answered the survey a certain way. Sometimes we use row or column totals to calculate relative frequencies. We call these *row relative frequencies* or *column relative frequencies*.

Below is the two-way frequency table for your reference. To calculate "the proportion of male students that prefer chocolate ice cream," divide the 22 male students who preferred chocolate ice cream by the total of 45 male students. This proportion is $\frac{22}{45} = 0.49$. Notice that you used the row total to make this calculation. This is a row relative frequency.

|         |        | **Favorite Ice Cream Flavor** | | | |
|---------|--------|-----------|------------|---------|-------|
|         |        | Chocolate | Strawberry | Vanilla | Total |
| **Gender** | Male   | 22        | 10         | 13      | 45    |
|         | Female | 8         | 15         | 14      | 37    |
|         | Total  | 30        | 25         | 27      | 82    |

## Exercises 18–22

In Exercise 13, you used the total number of students to calculate relative frequencies. These relative frequencies were the proportion of the whole group who answered the survey a certain way.

18. Suppose you are interested in the proportion of male students that prefer chocolate ice cream. How is this value different from "the proportion of students that are male and prefer chocolate ice cream"? Discuss this with your neighbor.

19. Use the table provided in Example 3 to calculate the following relative frequencies.

   a.   What proportion of students that prefer vanilla ice cream is female?

b.   What proportion of male students prefers strawberry ice cream?  Write a sentence explaining the meaning of this proportion in context of this problem.

c.   What proportion of female students prefers strawberry ice cream?

d.   What proportion of students who prefer strawberry ice cream is female?

20. A student is selected at random from this school.  What would you predict this student's favorite ice cream to be?  Explain why you choose this flavor.

21. Suppose the randomly selected student is male.  What would you predict his favorite flavor of ice cream to be?  Explain why you choose this flavor.

22. Suppose the randomly selected student is female.  What would you predict her favorite flavor of ice cream to be?  Explain why you choose this flavor.

**Lesson Summary**

- Univariate categorical data are displayed in a one-way frequency table.
- Bivariate categorical data are displayed in a two-way frequency table.
- *Relative frequency* is the frequency divided by a total (frequency/total).
- A *cell relative frequency* is a cell frequency divided by the total number of observations.
- A *row relative frequency* is a cell frequency divided by the row total.
- A *column relative frequency* is a cell frequency divided by the column total.

## Problem Set

Every student at Abigail Douglas Middle School is enrolled in exactly one extracurricular activity. The school counselor recorded data on extracurricular activity and gender for all 254 eighth-grade students at the school .

The counselor's findings for the 254 eighth-grade students are the following:

- Of the 80 students enrolled in band, 42 are male.
- Of the 21 students enrolled in art, 9 are female.
- Of the 65 students enrolled in choir, 20 are male.
- Of the 88 students enrolled in sports, 30 are female.

1. Complete the table below.

| | | Extracurricular Activities | | | | |
|---|---|---|---|---|---|---|
| | | Band | Choir | Sports | Art | Total |
| **Gender** | Female | | | | | |
| | Male | | | | | |
| | Total | | | | | |

2. Write a sentence explaining the meaning of the frequency 38 in this table.

3. What proportion of students is male and enrolled in choir?

4. What proportion of students is enrolled in a musical extracurricular activity (i.e., band or choir)?

5. What proportion of male students is enrolled in sports?

6.  What proportion of students enrolled in sports is male?

A pregnant woman will often undergo ultrasound tests to monitor her baby's health.  These tests can also be used to predict the gender of the baby, but these predictions are not always accurate.  Data on the gender predicted by ultrasound and the actual gender of the baby for 1,000 babies are summarized in the two-way table below.

|  |  | Predicted Gender | |
|---|---|---|---|
|  |  | Female | Male |
| Actual Gender | Female | 432 | 48 |
|  | Male | 130 | 390 |

7.  Write a sentence explaining the meaning of the frequency 130 in this table.

8.  What is the proportion of babies predicted to be male but were actually female?

9.  What is the proportion of incorrect ultrasound gender predictions?

10. For babies predicted to be female, what proportion of the predictions was correct?

11. For babies predicted to be male, what proportion of the predictions was correct?

COMMON CORE™    Lesson 13:   Summarizing Bivariate Categorical Data in a Two-Way Table
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# Lesson 14:  Association Between Categorical Variables

## Classwork

### Example 1

Suppose a random group of people are surveyed about their use of smartphones.  The results of the survey are summarized in the tables below.

**Smartphone Use and Gender**

|  | Use Smartphone | Do not Use Smartphone | Total |
|---|---|---|---|
| **Male** | 30 | 10 | 40 |
| **Female** | 45 | 15 | 60 |
| **Total** | 75 | 25 | 100 |

**Smartphone Use and Age**

|  | Use Smartphone | Do not Use Smartphone | Total |
|---|---|---|---|
| **Under 40 Years of Age** | 45 | 5 | 50 |
| **40 Years of Age or Older** | 30 | 20 | 50 |
| **Total** | 75 | 25 | 100 |

### Example 2

Suppose a sample of 400 participants (teachers and students) was randomly selected from the middle schools and high schools in a large city.  These participants responded to the question:

Which type of movie do you prefer to watch?

1.  Action (*The Avengers*, *Man of Steel*, etc.)
2.  Drama (*42 (The Jackie Robinson Story)*, *The Great Gatsby*, etc.)
3.  Science-Fiction (*Star Trek Into Darkness*, *World War Z*, etc.)
4.  Comedy (*Monsters University*, *Despicable Me 2*, etc.)

Movie preference and status (teacher/student) were recorded for each participant.

**Exercises 1–7**

1. Two variables were recorded. Are these variables categorical or numerical?

2. The results of the survey are summarized in the table below.

| | Movie Preference | | | | |
|---|---|---|---|---|---|
| | Action | Drama | Science-Fiction | Comedy | Total |
| **Student** | 120 | 60 | 30 | 90 | 300 |
| **Teacher** | 40 | 20 | 10 | 30 | 100 |
| **Total** | 160 | 80 | 40 | 120 | 400 |

   a. What proportion of participants who are teachers prefer action movies?

   b. What proportion of participants who are teachers prefer drama movies?

   c. What proportion of participants who are teachers prefer science-fiction movies?

   d. What proportion of participants who are teachers prefer comedy movies?

The answers to Exercise 2 are called row relative frequencies. Notice that you divided each cell frequency in the teacher row by the row total for that row. Below is a blank relative frequency table.

**Table of Row Relative Frequencies**

|  | Movie Preference | | | |
|---|---|---|---|---|
|  | Action | Drama | Science-Fiction | Comedy |
| **Student** |  |  |  |  |
| **Teacher** | (a) | (b) | (c) | (d) |

Write your answers from Exercise 2 in the indicated cells in the table above.

3. Find the row relative frequencies for the student row. Write your answers in the table above.
   a. What proportion of participants who are students prefers action movies?
   b. What proportion of participants who are students prefers drama movies?
   c. What proportion of participants who are students prefers science-fiction movies?
   d. What proportion of participants who are students prefers comedy movies?

4. Is a participant's status (i.e., teacher or student) related to what type of movie he or she would prefer to watch? Why or why not? Discuss this with your group.

5. What does it mean when we say that there is no association between two variables? Discuss this with your group.

6. Notice that the row relative frequencies for each movie type are the same for both the teacher and student rows. When this happens we say that the two variables, movie preference and status (student/teacher), are <u>NOT</u> associated. Another way of thinking about this is to say that knowing if a participant is a teacher (or a student) provides no information about his or her movie preference.

   What does it mean if row relative frequencies are not the same for all rows of a two-way table?

7.  You can also evaluate whether two variables are associated by looking at column relative frequencies instead of row relative frequencies. A column relative frequency is a cell frequency divided by the corresponding column total.  For example, the column relative frequency for the Student-Action cell is $\frac{120}{160} = 0.75$.

a.  Calculate the other column relative frequencies and write them in the table below.

**Table of Row Relative Frequencies**

|  | Movie Preference | | | |
|---|---|---|---|---|
|  | Action | Drama | Science-Fiction | Comedy |
| **Student** |  |  |  |  |
| **Teacher** |  |  |  |  |

b.  What do you notice about the column relative frequencies for the four columns?

c.  What would you conclude about association based on the column relative frequencies?

## Example 3

In the survey described in Example 2, gender for each of the 400 participants was also recorded.  Some results of the survey are given below:

- 160 participants preferred action movies.
- 80 participants preferred drama movies.
- 40 participants preferred science-fiction movies.
- 240 participants were females.
- 78 female participants preferred drama movies.
- 32 male participants preferred science-fiction movies.
- 60 female participants preferred action movies.

**Exercises 8–15**

Use the results from Example 3 to answer the following questions.  Be sure to discuss these questions with your group members.

8.  Complete the two-way frequency table that summarizes the data on movie preference and gender.

| | Movie Preference | | | | |
|---|---|---|---|---|---|
| | Action | Drama | Science-Fiction | Comedy | Total |
| **Student** | | | | | |
| **Teacher** | | | | | |
| **Total** | | | | | |

9.  What proportion of the participants is female?

10. If there were no association between gender and movie preference, should you expect more females than males or fewer females than males to prefer action movies?  Explain.

11. Make a table of row relative frequencies of each movie type for the male row and the female row.  Refer to Exercises 2–4 to review how to complete the table below.

| | Movie Preference | | | |
|---|---|---|---|---|
| | Action | Drama | Science-Fiction | Comedy |
| **Student** | | | | |
| **Teacher** | | | | |

Suppose that you randomly pick 1 of the 400 participants. Use the table of row relative frequencies above to answer the following questions.

12. If you had to predict what type of movie this person chose, what would you predict? Explain why you made this choice.

13. If you know that the randomly selected participant is female, would you predict that her favorite type of movie was action? If not, what would you predict and why?

14. If knowing the value of one of the variables provides information about the value of the other variable, then there is an association between the two variables.

    Is there an association between the variables gender and movie preference? Explain.

15. So what can be said when two variables are associated? Read the following sentences. Decide if the sentence is a correct statement based upon the survey data. If it is not correct, explain why not.

    a.  More females than males participated in the survey.

    b.  Males tend to prefer action and science-fiction moves.

    c.  Being female causes one to prefer drama movies.

**Lesson Summary**

- Saying that two variables ARE NOT associated means that knowing the value of one variable provides no information about the value of the other variable.

- Saying that two variables ARE associated means that knowing the value of one variable provides information about the value of the other variable.

- To determine if two variables are associated, calculate row relative frequencies. If the row relative frequencies are about the same for all of the rows, it is reasonable to say that there is no association between the two variables that define the table.

- Another way to decide if there is an association between two categorical variables is to calculate column relative frequencies. If the column relative frequencies are about the same for all of the columns, it is reasonable to say that there is no association between the two variables that define the table.

- If the row relative frequencies are quite different for some of the rows, it is reasonable to say that there is an association between the two variables that define the table.

## Problem Set

A sample of 200 middle school students was randomly selected from the middle schools in a large city. Answers to several survey questions were recorded for each student. The tables below summarize the results of the survey.

For each table, calculate the row relative frequencies for the female row and for the male row. Write the row relative frequencies beside the corresponding frequencies in each table below.

1. This table summarizes the results of the survey data for the two variables, gender and which sport the students prefer to play. Is there an association between gender and which sport the students prefer to play? Explain.

| | | Sport | | | | |
|---|---|---|---|---|---|---|
| | | Football | Basketball | Volleyball | Soccer | Total |
| **Gender** | Female | 2 | 29 | 28 | 38 | 97 |
| | Male | 35 | 26 | 8 | 24 | 103 |
| | Total | 37 | 65 | 36 | 62 | 200 |

2.  This table summarizes the results of the survey data for the two variables, gender and the students' T-shirt sizes.  Is there an association between gender and T-shirt size?  Explain.

|  |  | School T-Shirt Sizes | | | | |
|---|---|---|---|---|---|---|
|  |  | Small | Medium | Large | X-Large | Total |
| **Gender** | Female | 47 | 35 | 13 | 2 | 97 |
|  | Male | 11 | 41 | 42 | 9 | 103 |
|  | Total | 58 | 76 | 55 | 11 | 200 |

3.  This table summarizes the results of the survey data for the two variables, gender and favorite type of music.  Is there an association between gender and favorite type of music?  Explain

|  |  | Favorite Type of Music | | | | |
|---|---|---|---|---|---|---|
|  |  | Pop | Hip Hop | Alternative | Country | Total |
| **Gender** | Female | 35 | 28 | 11 | 23 | 97 |
|  | Male | 37 | 30 | 13 | 23 | 103 |
|  | Total | 72 | 58 | 24 | 46 | 200 |