Lesson 7: Infinite Decimals

Classwork

Opening Exercise

a. Write the expanded form of the decimal 0.3765 using powers of 10.

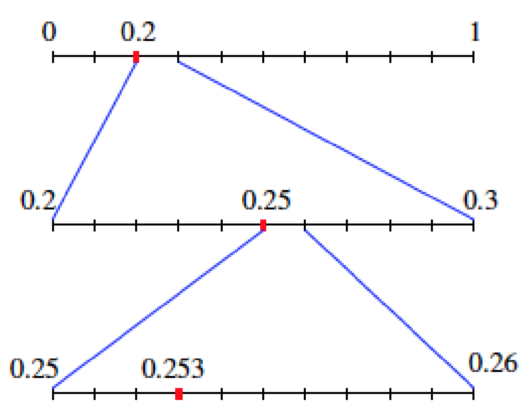
b. Write the expanded form of the decimal $0.3333333\ldots$ using powers of 10.

c. What is an infinite decimal? Give an example.

d. Do you think it is acceptable to write that $1 = 0.99999\ldots$? Why or why not?

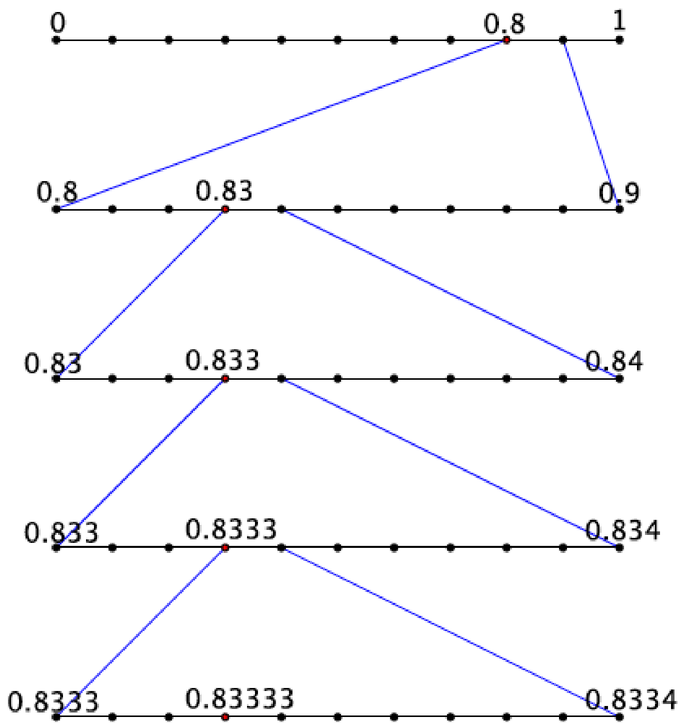
Example 1

The number 0.253 is represented on the number line below.



Example 2

The number $\frac{5}{6} = 0.833333\ldots = 0.8\overline{3}$ is represented on the number line below.

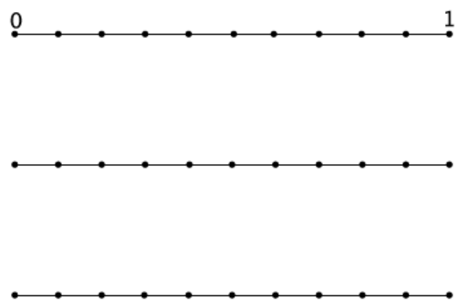


Exercises 1–6

1.

a. Write the expanded form of the decimal 0.125 using powers of 10.

b. Show on the number line the representation of the decimal 0.125.

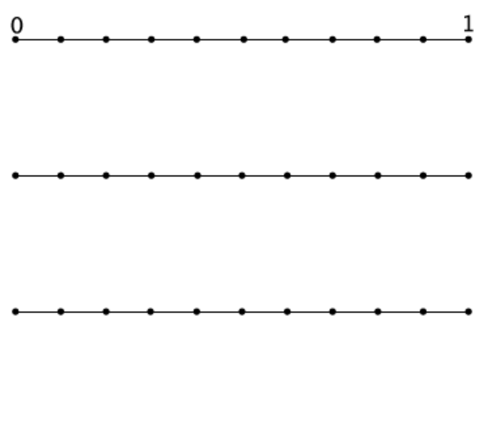


c. Is the decimal finite or infinite? How do you know?

2.

a. Write the expanded form of the decimal 0.3875 using powers of 10.

b. Show on the number line the representation of the decimal 0.3875.

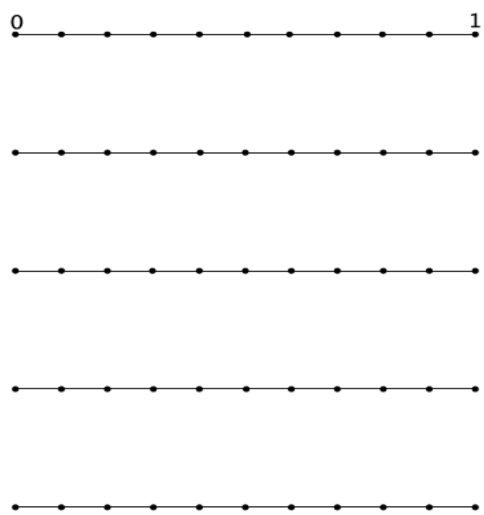


c. Is the decimal finite or infinite? How do you know?

3.

a. Write the expanded form of the decimal $0.777777\ldots$ using powers of 10.

b. Show on the number line the representation of the decimal $0.777777\ldots$.

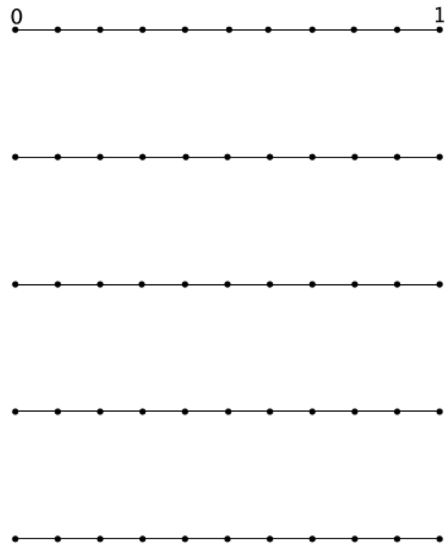


c. Is the decimal finite or infinite? How do you know?

4.

a. Write the expanded form of the decimal $0.\overline{45}$ using powers of 10.

b. Show on the number line the representation of the decimal $0.\overline{45}$.



c. Is the decimal finite or infinite? How do you know?

5. Order the following numbers from least to greatest: 2.121212, 2.1, 2.2, and $2.\overline{12}$.

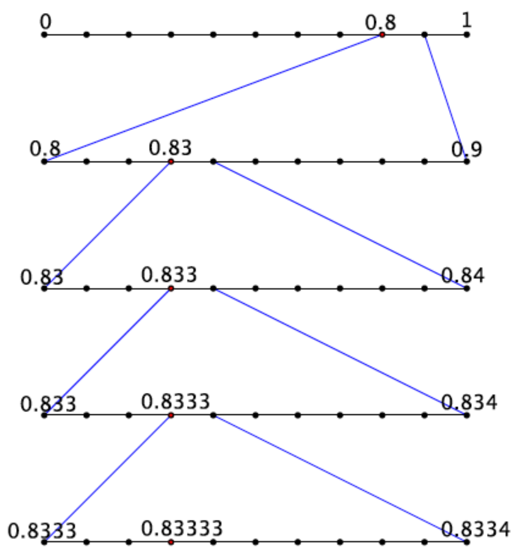
6. Explain how you knew which order to put the numbers in.

Lesson Summary

An infinite decimal is a decimal whose expanded form and number line representation are infinite.

Example:
The expanded form of the decimal $0.83333\ldots$ is $0.8\overline{3} = \frac{8}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \frac{3}{10^4} + \cdots$.

The number is represented on the number line shown below. Each new line is a magnification of the interval shown above it. For example, the first line is the unit from 0 to 1 divided into ten equal parts, or tenths. The second line is the interval from 0.8 to 0.9 divided into ten equal parts, or hundredths. The third line is the interval from 0.83 to 0.84 divided into ten equal parts, or thousandths, and so on.

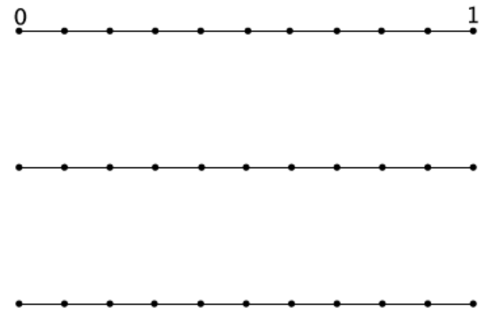


With each new line, we are representing an increasingly smaller value of the number, so small that the amount approaches a value of zero. Consider the 20^{th} line of the picture above. We would be adding $\frac{3}{10^{20}}$ to the value of the number, which is 0.00000000000000000003. It should be clear that $\frac{3}{10^{20}}$ is a very small number and is fairly close to a value of zero.

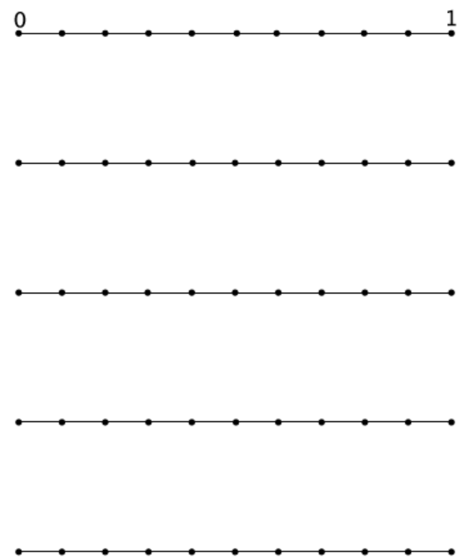
This reasoning is what we use to explain why the value of the infinite decimal $0.\overline{9}$ is 1.

Problem Set

1.
   a. Write the expanded form of the decimal 0.625 using powers of 10.
   b. Show on the number line the representation of the decimal 0.625.
   c. Is the decimal finite or infinite? How do you know?



2.
   a. Write the expanded form of the decimal $0.\overline{370}$ using powers of 10.
   b. Show on the number line the representation of the decimal $0.370370\ldots$.
   c. Is the decimal finite or infinite? How do you know?



3. Which is a more accurate representation of the number $\frac{2}{3}$: 0.6666 or $0.\overline{6}$? Explain. Which would you prefer to compute with?

4. Explain why we shorten infinite decimals to finite decimals to perform operations. Explain the effect of shortening an infinite decimal on our answers.

5. A classmate missed the discussion about why $0.\overline{9} = 1$. Convince your classmate that this equality is true.

6. Explain why $0.3333 < 0.33333$.