## Lesson 1: Wishful Thinking—Does Linearity Hold?

## Classwork

## Exercises 1-2

Look at these common mistakes that students make, and answer the questions that follow.

1. If $f(x)=\sqrt{x}$, does $f(a+b)=f(a)+f(b)$, when $a$ and $b$ are not negative?
a. Can we find a counterexample to refute the claim that $f(a+b)=f(a)+f(b)$ for all nonnegative values of $a$ and $b$ ?
b. Find some nonnegative values for $a$ and $b$ for which the statement, by coincidence, happens to be true.
c. Find all values of $a$ and $b$ for which the statement is true. Explain your work and the results.
d. Why was it necessary for us to consider only nonnegative values of $a$ and $b$ ?
e. Does $f(x)=\sqrt{x}$ display ideal linear properties? Explain.
2. If $f(x)=x^{3}$, does $f(a+b)=f(a)+f(b)$ ?
a. Substitute in some values of $a$ and $b$ to show this statement is not true in general.
b. Find some values for $a$ and $b$ for which the statement, by coincidence, happens to work.
c. Find all values of $a$ and $b$ for which the statement is true. Explain your work and the results.
d. Is this true for all positive and negative values of $a$ and $b$ ? Explain and prove by choosing positive and negative values for the variables.
e. Does $f(x)=x^{3}$ display ideal linear properties? Explain.

## Problem Set

Study the statements given in Problems 1-3. Prove that each statement is false, and then find all values of $a$ and $b$ for which the statement is true. Explain your work and the results.

1. If $f(x)=x^{2}$, does $f(a+b)=f(a)+f(b)$ ?
2. If $f(x)=x^{\frac{1}{3}}$, does $f(a+b)=f(a)+f(b)$ ?
3. If $f(x)=\sqrt{4 x}$, does $f(a+b)=f(a)+f(b)$ ?
4. Think back to some mistakes that you have made in the past simplifying or expanding functions. Write the statement that you assumed was correct that was not, and find numbers that prove your assumption was false.
