## Lesson 2: Wishful Thinking—Does Linearity Hold?

## Classwork

## Exercises 1-5

1. Let $f(x)=\sin x$. Does $f(2 x)=2 f(x)$ for all values of $x$ ? Is it true for any values of $x$ ? Show work to justify your answer.
2. Let $f(x)=\log (x)$. Find a value for $a$ such that $f(2 a)=2 f(a)$. Is there one? Show work to justify your answer.
3. Let $f(x)=10^{x}$. Show that $f(a+b)=f(a)+f(b)$ is true for $a=b=\log (2)$ and that it is not true for $a=b=2$.
4. Let $f(x)=\frac{1}{x}$. Are there any real numbers $a$ and $b$ so that $f(a+b)=f(a)+f(b)$ ? Explain.
5. What do your findings from these Exercises illustrate about the linearity of these functions? Explain.

## Problem Set

Examine the equations given in Problems 1-4, and show that the functions $f(x)=\cos x$ and $f(x)=\tan x$ are not linear transformations by demonstrating that they do not satisfy the conditions indicated for all real numbers. Then, find values of $x$ and/or $y$ for which the statement holds true.

1. $\cos (x+y)=\cos (x)+\cos (y)$
2. $\cos (2 x)=2 \cos (x)$
3. $\tan (x+y)=\tan (x)+\tan (y)$
4. $\tan (2 x)=2 \tan (x)$
5. Let $f(x)=\frac{1}{x^{2}}$, are there any real numbers $a$ and $b$ so that $f(a+b)=f(a)+f(b)$ ? Explain.
6. Let $f(x)=\log x$, find values of $a$ such that $f(3 a)=3 f(a)$.
7. Let $f(x)=\log x$, find values of $a$ such that $f(k a)=k f(a)$.
8. Based on your results from the previous two problems, form a conjecture about whether $f(x)=\log x$ represents a linear transformation.
9. Let $f(x)=a x^{2}+b x+c$.
a. Describe the set of all values for $a, b$, and $c$ that make $f(x+y)=f(x)+f(y)$ valid for all real numbers $x$ and $y$.
b. What does your result indicate about the linearity of quadratic functions?

Trigonometry Table

| Angles Measure <br> $(x$ degrees $)$ | Angle Measure <br> $(x$ radians $)$ | $\sin (x)$ | $\boldsymbol{\operatorname { c o s } ( x )}$ |
| :---: | :---: | :---: | :---: |
| 0 |  |  |  |
| 30 | $\frac{\pi}{4}$ |  |  |
|  | $\frac{\pi}{3}$ |  |  |
| 90 |  |  |  |
|  |  |  |  |

