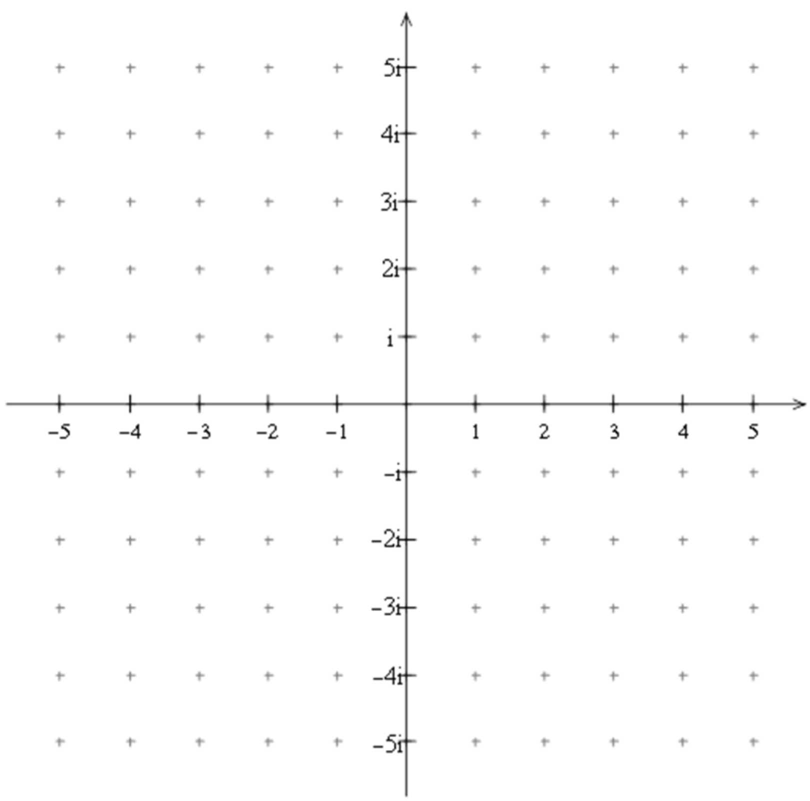
Lesson 5: An Appearance of Complex Numbers

Classwork

Opening Exercise

Write down two fundamental facts about i that you learned in the previous lesson.

Discussion: Visualizing Complex Numbers



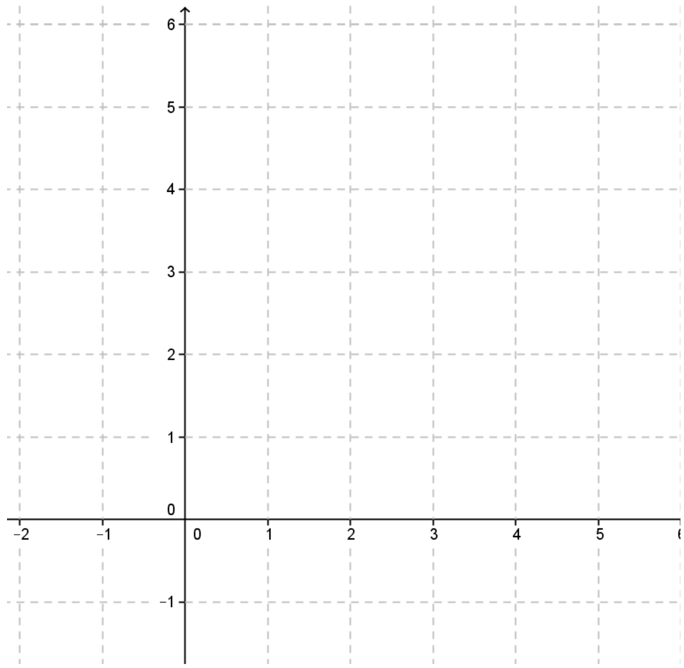
Exercises

1. Give an example of a real number, an imaginary number, and a complex number. Use examples that have not already been discussed in the lesson.

2. In the complex plane, what is the horizontal axis used for? What is the vertical axis used for?

3. How would you represent $-4 + 3i$ in the complex plane?

For Exercises 4–7, let $a = 1 + 3i$ and $b = 2 - i$.



4. Find $a + b$. Then plot a, b, and $a + b$ in the complex plane.

5. Find $a - b$. Then plot a, b, and $a - b$ in the complex plane.

6. Find $2a$. Then plot a and $2a$ in the complex plane.

7. Find $a \cdot b$. Then plot a, b, and $a \cdot b$ in the complex plane.

Problem Set

1. The number 5 is a real number. Is it also a complex number? Try to find values of a and b so that $5 = a + bi$.

2. The number $3i$ is an imaginary number and a multiple of i. Is it also a complex number? Try to find values of a and b so that $3i = a + bi$.

3. Daria says that "every real number is a complex number." Do you agree with her? Why or why not?

4. Colby says that "every imaginary number is a complex number." Do you agree with him? Why or why not?

In Problems 5–9, perform the indicated operations. Report each answer as a complex number $w = a + bi$, and graph it in a complex plane.

5. Given $z_1 = -9 + 5i$, $z_2 = -10 - 2i$, find $\mathrm{w} = z_1 + z_2$, and graph z_1, z_2, and w.

6. Given $z_1 = -4 + 10i$,$z_2 = -7 - 6i$, find $\mathrm{w} = z_1 - z_2$, and graph z_1, z_2, and w.

7. Given $z_1 = 3\sqrt{2} + 2i$, $z_2 = \sqrt{2} - i$, find $\mathrm{w} = z_1 - z_2$, and graph z_1, z_2, and w.

8. Given $z_1 = 3$, $z_2 = -4 + 8i$, find $\mathrm{w} = z_1 \cdot z_2$, and graph z_1, z_2, and w.

9. Given $z_1 = \frac{1}{4}$, $z_2 = 12 - 4i$, find $\mathrm{w} = z_1 \cdot z_2$, and graph z_1, z_2, and w.

10. Given $z_1 = -1$, $z_2 = 3 + 4i$, find $\mathrm{w} = z_1 \cdot z_2$, and graph z_1, z_2, and w.

11. Given $z_1 = 5 + 3i$, $z_2 = -4 - 2i$, find $\mathrm{w} = z_1 \cdot z_2$, and graph z_1, z_2, and w.

12. Given $z_1 = 1 + i$, $z_2 = 1 + i$, find $\mathrm{w} = z_1 \cdot z_2$, and graph z_1, z_2, and w.

13. Given $z_1 = 3$, $z_2 = i$, find $\mathrm{w} = z_1 \cdot z_2$, and graph z_1, z_2, and w.

14. Given $z_1 = 4 + 3i$, $z_2 = i$, find $\mathrm{w} = z_1 \cdot z_2$, and graph z_1, z_2, and w.

15. Given $z_1 = 2\sqrt{2} + 2\sqrt{2}i$, $z_2 = -\sqrt{2} + \sqrt{2}i$, find $\mathrm{w} = z_1 \cdot z_2$, and graph z_1, z_2, and w.

16. Represent $w = -4 + 3i$ as a point in the complex plane.

17. Represent $2w$ as a point in the complex plane. $2w = 2(-4 + 3i) = -8 + 6i$

18. Compare the positions of w and $2w$ from Problems 10 and 11. Describe what you see. (Hint: Draw a segment from the origin to each point.)