Lesson 10: The Geometric Effect of Some Complex Arithmetic

Classwork

Opening Exercises

1. Given $z = 3 - 2i$, plot and label the following and describe the geometric effect of the operation.
   a. z

   b. $z - 2$

   c. $z + 4i$

   d. $z + (-2 + 4i)$

2. Describe the geometric effect of the following:
   a. Multiplying by i.

   b. Taking the complex conjugate.

   c. What operation reflects a complex number across the imaginary axis?

Example 1

Plot the given points, then plot the image $L(z) = 2z$.

a. $z_1 = 3$

b. $z_2 = 2i$

c. $z_3 = 1 + i$

d. $z_4 = -4 + 3i$

e. $z_5 = 2 - 5i$

Exercises 1–7

Plot the given points, then plot the image $L(z) = iz$.

1. $z_1 = 3$

2. $z_2 = 2i$

3. $z_3 = 1 + i$

4. $z_4 = -4 + 3i$

5. $z_5 = 2 - 5i$

6. What is the geometric effect of the transformation? Confirm your conjecture using the slope of the segment joining the origin to the point and then to its image.

7. Is $L(z)$ a linear transformation? Explain how you know.

Example 2

Describe the geometric effect of $L(z) = (1 + i)z$ given the following. Plot the images on graph paper, and describe the geometric effect in words.

a. $z_1 = 1$

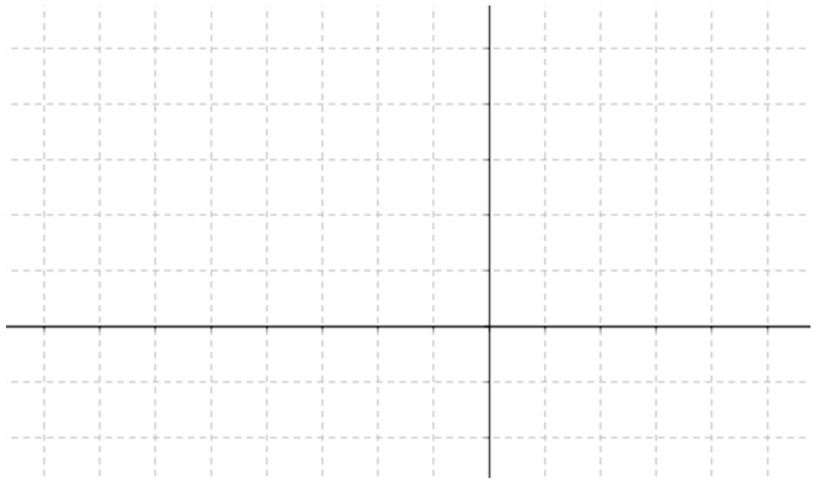
b. $z_2 = i$

c. $z_4 = 1 + i$

d. $z_5 = 4 + 6i$

Problem Set

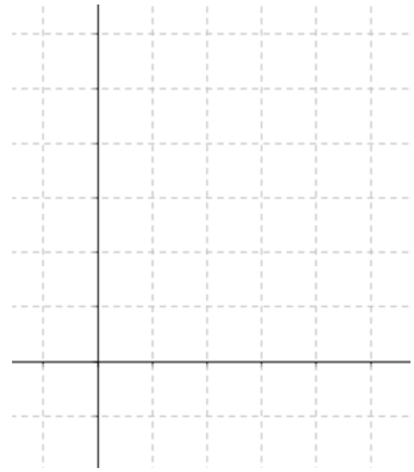
1. Let $z = -4 + 2i$, simplify the following and describe the geometric effect of the operation. Plot the result in the complex plane.
   a. $z + 2 - 3i$
   b. $z - 2 - 3i$
   c. $z - (2 - 3i)$
   d. $2z$
   e. $\frac{z}{2}$



2. Let $z = 1 + 2i$, simplify the following and describe the geometric effect of the operation.
   a. iz
   b. $i^2 z$
   c. $\bar{z}$
   d. $-\bar{z}$
   e. $i\bar{z}$
   f. $2iz$
   g. $iz + 5 - 3i$

3. Simplify the following expressions.
   a. $(4 - 2i)(5 - 3i)$
   b. $(-2 + 3i)(-2 - 3i)$
   c. $(1 + i)^2$
   d. $(1 + i)^{10}$ (Hint: $b^{nm} = (b^n)^m$)
   e. $\frac{-1 + 2i}{1 - 2i}$
   f. $\frac{x^2 + 4}{x - 2i}$, provided $x \neq 2i$.

4. Given $z = 2 + i$, describe the geometric effect of the following. Plot the result.
   a. $z(1+i)$
   b. $z\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$



5. We learned that multiplying by i produces a $90°$ counterclockwise rotation about the origin. What do we need to multiply by to produce a $90°$ clockwise rotation about the origin?

6. Given z is a complex number $a + bi$, determine if $L(z)$ is a linear transformation. Explain why or why not.
   a. $L(z) = i^3 z$
   b. $L(z) = z + 4i$