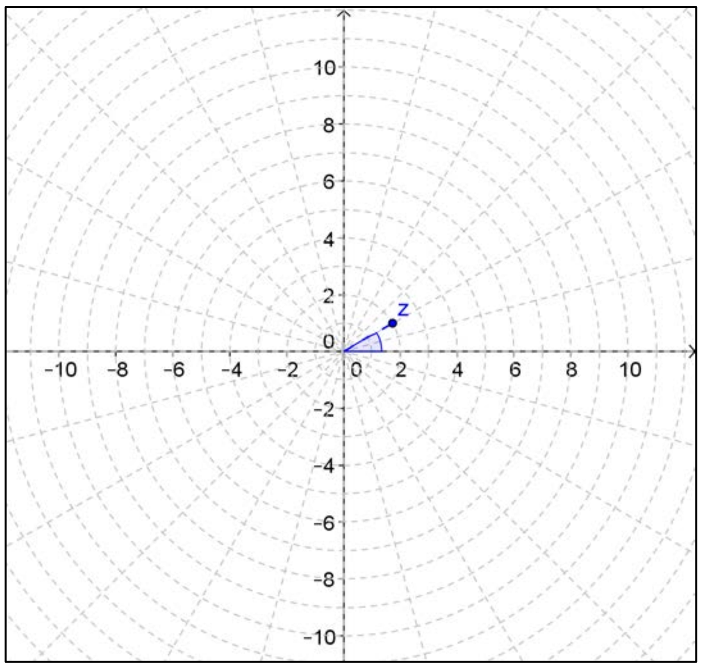
Lesson 19: Exploiting the Connection to Trigonometry

Classwork

Opening Exercise

A polar grid is shown below. The grid is formed by rays from the origin at equal rotation intervals and concentric circles centered at the origin. The complex number $z = \sqrt{3} + i$ is graphed on this polar grid.



a. Use the polar grid to identify the modulus and argument of z.

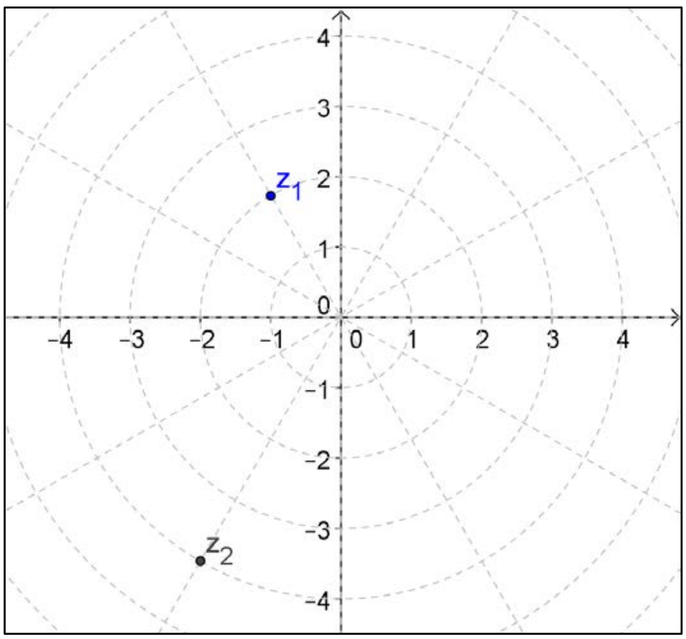
b. Graph the next three powers of z on the polar grid. Explain how you got your answers.

c. Write the polar form of the number in the table below, and then rewrite it in rectangular form.

Power of z	Polar Form	Rectangular Form
$\sqrt{3}+i$		
$\left(\sqrt{3}+i\right)^2$		
$\left(\sqrt{3}+i\right)^3$		
$\left(\sqrt{3}+i\right)^4$		

Exercises 1–3

The complex numbers $z_2 = \left(-1+\sqrt{3}i\right)^2$ and z_1 are graphed below.



1. Use the graph to help you write the numbers in polar and rectangular form.

2. Describe how the modulus and argument of $z_1 = -1+\sqrt{3i}$ are related to the modulus and argument of $z_2 = \left(-1+\sqrt{3}i\right)^2$.

3. Why could we call $-1 + \sqrt{3}i$ a square root of $-2 - 2\sqrt{3}i$?

Example 1: Find the Two Square Roots of a Complex Number

Find both of the square roots of $-2 - 2\sqrt{3}i$.

Exercises 4–6

4. Find the cube roots of $-2 = 2\sqrt{3}i$.

5. Find the square roots of $4i$.

6. Find the cube roots of 8.

Lesson Summary

Given a complex number z with modulus r and argument θ, the n^{th} roots of z are given by

$$\sqrt[n]{r}\left(\cos\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)+i\sin\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)\right)$$

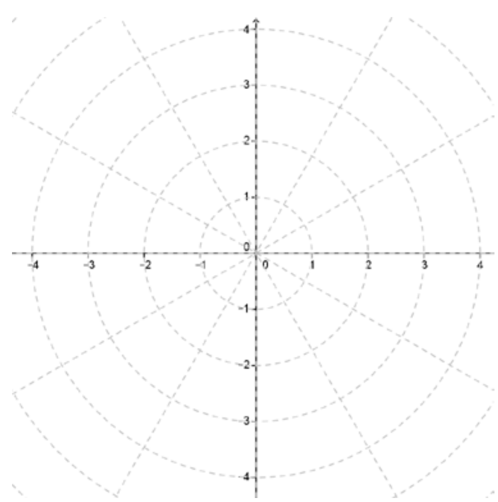
for integers k and n such that $n > 0$ and $0 \leq k < n$.

Problem Set

1. For each complex number what is z^2?
   a. $1+\sqrt{3}i$
   b. $3-3i$
   c. $4i$
   d. $-\frac{\sqrt{3}}{2}+\frac{1}{2}i$
   e. $\frac{1}{9}+\frac{1}{9}i$
   f. -1

2. For each complex number, what are the square roots of z?
   a. $1+\sqrt{3}i$
   b. $3-3i$
   c. $4i$
   d. $-\frac{\sqrt{3}}{2}+\frac{1}{2}i$
   e. $\frac{1}{9}+\frac{1}{9}i$
   f. -1

3. For each complex number, graph z, z^2, and z^3 on a polar grid.
   a. $2\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)$
   b. $3(\cos(210°)+i\sin(210°))$
   c. $2\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)$
   d. $\cos(\pi)+i\sin(\pi)$
   e. $4\left(\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right)$
   f. $\frac{1}{2}(\cos(60°)+i\sin(60°))$



4. What are the cube roots of $-3i$?

5. What are the fourth roots of 64?

6. What are the square roots of $-4-4i$?

7. Find the square roots of -5. Show that the square roots satisfy the equation $x^2+5=0$.

8. Find the cube roots of 27. Show that the cube roots satisfy the equation $x^3-27=0$.