Lesson 22: Modeling Video Game Motion with Matrices

Classwork

Opening Exercise

Let $D\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$.

a. Plot the point $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

b. Find $D\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, and plot it.

c. Describe the geometric effect of performing the transformation $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow D\begin{pmatrix} x \\ y \end{pmatrix}$.

Exercises 1–2

1. Let $f(t) = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix}\begin{pmatrix} 2 \\ 4 \end{pmatrix}$, where t represents time, measured in seconds. $P = f(t)$ represents the position of a moving object at time t. If the object starts at the origin, how long would it take to reach $(12, 24)$?

2. Let $g(t) = \begin{pmatrix} kt & 0 \\ 0 & kt \end{pmatrix}\begin{pmatrix} 2 \\ 4 \end{pmatrix}$.

   a. Find the value of k that moves an object from the origin to $(12,24)$ in just 2 seconds.

   b. Find the value of k that moves an object from the origin to $(12, 24)$ in 30 seconds.

Exercises 3–4

3. Let $f(t) = \begin{pmatrix} 2+t & 0 \\ 0 & 2+t \end{pmatrix}\begin{pmatrix} 5 \\ 7 \end{pmatrix}$, where t represents time, measured in seconds, and $f(t)$ represents the position of a moving object at time t.
   a. Find the position of the object at $t = 0, t = 1$, and $t = 2$.

   b. Write $f(t)$ in the form $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$.

4. Write the transformation $g(t) = \begin{pmatrix} 15+5t \\ -6-2t \end{pmatrix}$ as a matrix transformation.

Exercise 5

5. An object is moving in a straight line from $(18,12)$ to the origin over a 6-second period of time. Find a function $f(t)$ that gives the position of the object after t seconds. Write your answer in the form $f(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$, then express $f(t)$ as a matrix transformation.

Exercises 6–9

6. Write a rule for the function that shifts every point in the plane 6 units to the left.

7. Write a rule for the function that shifts every point in the plane 9 units upward.

8. Write a rule for the function that shifts every point in the plane 10 units down and 4 units to the right.

9. Consider the rule $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x - 7 \\ y + 2 \end{pmatrix}$. Describe the effect this transformation has on the plane.

Problem Set

1. Let $D\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ find and plot the following.
   a. Plot the point: $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and find $D\begin{pmatrix} -1 \\ 2 \end{pmatrix}$, and plot it.
   b. Plot the point: $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and find $D\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, and plot it.
   c. Plot the point: $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and find $D\begin{pmatrix} 5 \\ 2 \end{pmatrix}$, and plot it.

2. Let $f(t) = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$, find $f(0), f(1), f(2), f(3)$, and plot them on the same graph.

3. Let $f(t) = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ represent the location of an object at time t that is measured in seconds.
   a. How long does it take the object to travel from the origin to the point $\begin{pmatrix} 12 \\ 8 \end{pmatrix}$?
   b. Find the speed of the object in the horizontal direction and in the vertical direction.

4. Let $f(t) = \begin{pmatrix} 0.2t & 0 \\ 0 & 0.2t \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix}, h(t) = \begin{pmatrix} 2t & 0 \\ 0 & 2t \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Which one will reach the point $\begin{pmatrix} 12 \\ 8 \end{pmatrix}$ first? The time t is measured in seconds.

5. Let $f(t) = \begin{pmatrix} kt & 0 \\ 0 & kt \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, find the value of k that moves the object from the origin to $\begin{pmatrix} -45 \\ -30 \end{pmatrix}$ in 5 seconds.

6. Write $f(t)$ in the form $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ if
   a. $f(t) = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix}$.
   b. $f(t) = \begin{pmatrix} 2t+1 & 0 \\ 0 & 2t+1 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
   c. $f(t) = \begin{pmatrix} \frac{t}{2} - 3 & 0 \\ 0 & \frac{t}{2} - 3 \end{pmatrix}\begin{pmatrix} 4 \\ -6 \end{pmatrix}$.

7. Let $f(t) = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ represent the location of an object after t seconds.
   a. If the object starts at $\begin{pmatrix} 6 \\ 15 \end{pmatrix}$, how long would it take to reach $\begin{pmatrix} 34 \\ 85 \end{pmatrix}$?
   b. Write the new function $f(t)$ that gives the position of the object after t seconds.
   c. Write $f(t)$ as a matrix transformation.

8. Write the following functions as a matrix transformation.

   a. $f(t) = \begin{pmatrix} 10 + 2t \\ 15 + 3t \end{pmatrix}$

   b. $f(t) = \begin{pmatrix} -6t + 15 \\ 8t - 20 \end{pmatrix}$

9. Write a function rule that represents the change in position of the point $\begin{pmatrix} x \\ y \end{pmatrix}$ for the following.

   a. 5 units to the right and 3 units downward.

   b. 2 units downward and 3 units to the left

   c. 3 units upward, 5 units to the left, and then it dilates by 2

   d. 3 units upward, 5 units to the left, and then it rotates by $\frac{\pi}{2}$ counterclockwise.

10. Annie is designing a video game and wants her main character to be able to move from any given point $\begin{pmatrix} x \\ y \end{pmatrix}$ in the following ways: right 1 unit, jump up 1 unit, and both jump up and move right 1 unit each.

    a. What function rules can she use to represent each time the character moves?

    b. Annie is also developing a ski slope stage for her game and wants to model her character's position using matrix transformations. Annie wants the player to start at $\begin{pmatrix} -20 \\ 10 \end{pmatrix}$ and eventually pass through the origin moving 5 units per second down. How fast does the player need to move to the right in order to pass through the origin? What matrix transformation can Annie use to describe the movement of the character? If the far-right of the screen is at $x = 20$, how long until the player moves off the screen traveling this path?

11. Remy thinks that he has developed matrix transformations to model the movements of Annie's characters in Problem 10 from any given point $\begin{pmatrix} x \\ y \end{pmatrix}$, and he has tested them on the point $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. This is the work Remy did on the transformations:

    $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad$ $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\quad$ $\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

    Do these matrix transformations accomplish the movements that Annie wants to program into the game? Explain why or why not.

12. Nolan has been working on how to know when the path of a point can be described with matrix transformations and how to know when it requires translations and cannot be described with matrix transformations. So far he has been focusing on the following two functions which both pass through the point $(2,5)$:

    $f(t) = \begin{pmatrix} 2t + 6 \\ 5t + 15 \end{pmatrix}$ and $g(t) = \begin{pmatrix} t + 2 \\ t + 5 \end{pmatrix}$

    a. If we simplify these functions algebraically, how does the rule for f differ from the rule for g? What does this say about which function can be expressed with matrix transformations?

    b. Nolan has noticed functions that can be expressed with matrix transformations always pass through the origin; does either f or g pass through the origin, and does this support or contradict Nolan's reasoning?

    c. Summarize the results of parts (a) and (b) to describe how we can tell from the equation for a function or from the graph of a function that it can be expressed with matrix transformations.