

## Lesson 24: Matrix Notation Encompasses New Transformations!

### Classwork

#### Example 1

Determine the following:

a.  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

b.  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -7 \\ 12 \end{bmatrix}$

c.  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ -2 & 1 \end{bmatrix}$

d.  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -3 \\ -7 & 6 \end{bmatrix}$

e.  $\begin{bmatrix} 9 & 12 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

f.  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

g.  $\begin{bmatrix} x & y \\ z & w \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**Example 2**

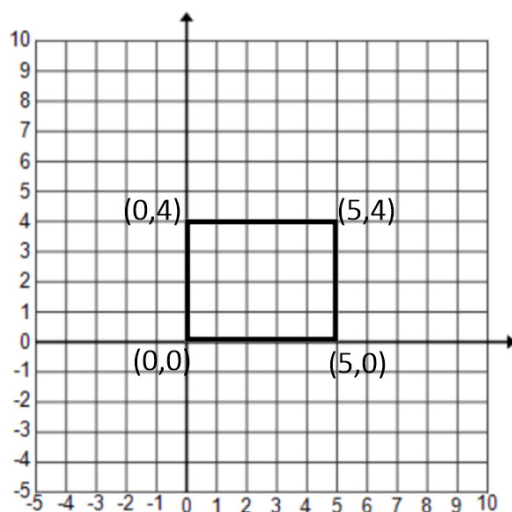
Can the reflection about the real axis  $L(z) = \bar{z}$  be expressed in matrix notation?

**Exercises 1–3**

1. Express a reflection about the vertical axis in matrix notation. Prove that it produces the desired reflection by using matrix multiplication.
2. Express a reflection about both the horizontal and vertical axes in matrix notation. Prove that it produces the desired reflection by using matrix multiplication.
3. Express a reflection about the vertical axis and a dilation with a scale factor of 6 in matrix notation. Prove that it produces the desired reflection by using matrix multiplication.

### Exercises 4–8

Explore the transformation given by each matrix below. Use the graph of the rectangle provided to assist in the exploration. Describe the effect on the graph of the rectangle, and then show the general effect of the transformation by using matrix multiplication.



Matrix	Transformation of the rectangle	General effect of the matrix
4. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$		
5. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$		
6. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$		
7. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$		
8. $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$		

**Lesson Summary**

All matrices in the form  $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$  correspond to a transformation of some kind.

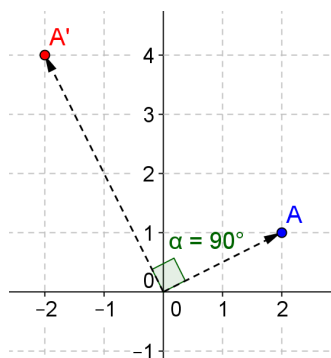
- The matrix  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  reflects all coordinates about the horizontal axis.
- The matrix  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$  reflects all coordinates about the vertical axis.
- The matrix  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  is the identity matrix and corresponds to a transformation that leaves points alone.
- The matrix  $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$  is the zero matrix and corresponds to a dilation of scale factor 0.

**Problem Set**

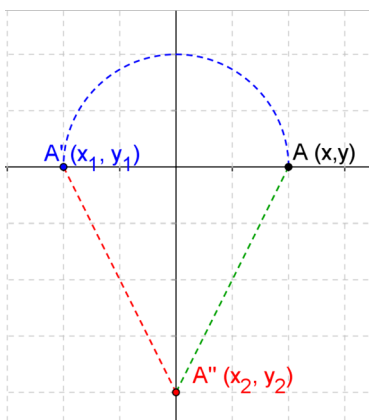
1. What matrix do you need to use to reflect the following points about the  $y$ -axis? What is the resulting matrix when this is done? Show all work and sketch it.
  - a.  $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$
  - b.  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$
  - c.  $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$
  - d.  $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$
  - e.  $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$
  - f.  $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$
2. What matrix do you need to use to reflect the following points about the  $x$ -axis? What is the resulting matrix when this is done? Show all work and sketch it.
  - a.  $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$
  - b.  $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
  - c.  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$
  - d.  $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$
  - e.  $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$
  - f.  $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

3. What matrix do you need to use to dilate the following points by a given factor? What is the resulting matrix when this is done? Show all work and sketch it.
- $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ , a factor of 3
  - $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ , a factor of 2
  - $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ , a factor of 1
  - $\begin{pmatrix} -4 \\ -6 \end{pmatrix}$ , a factor of  $\frac{1}{2}$
  - $\begin{pmatrix} 9 \\ 3 \end{pmatrix}$ , a factor of  $\frac{1}{3}$
  - $\begin{pmatrix} \sqrt{3} \\ \sqrt{11} \end{pmatrix}$ , a factor of  $\sqrt{2}$
4. What matrix will rotate the given point by the angle? What is the resulting matrix when this is done? Show all work and sketch it.
- $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\frac{\pi}{2}$  radians
  - $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\frac{\pi}{3}$  radians
  - $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\frac{\pi}{6}$  radians
  - $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\frac{\pi}{4}$  radians
  - $\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$ ,  $\frac{\pi}{6}$  radians
  - $\begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$ ,  $\frac{\pi}{4}$  radians
  - $\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$ ,  $\pi$  radians
  - $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $-\frac{\pi}{6}$  radians

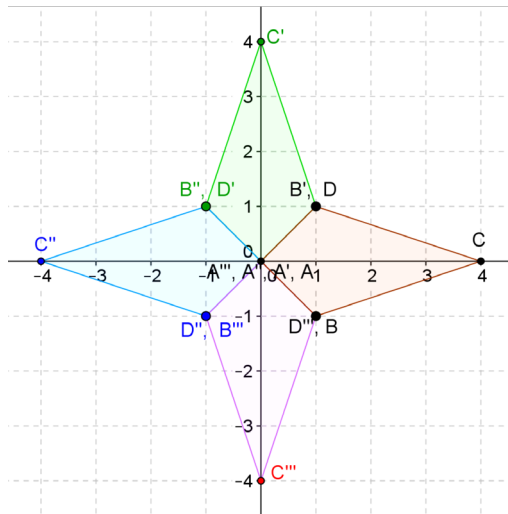
5. For the transformation shown below, find the matrix that will transform point  $A$  to  $A'$ , and verify your answer.



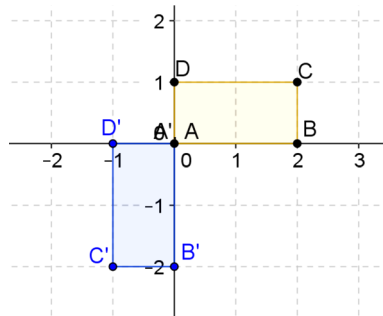
6. In this lesson, we learned  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  will produce a reflection about the line  $y = x$ . What matrix will produce a reflection about the line  $y = -x$ ? Verify your answers by testing the given point  $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$  and graphing them on the coordinate plane.
7. Describe the transformation and the translations in the diagram below. Write the matrices that will perform the tasks. What is the area that these transformations and translations have enclosed?



8. Given the kite figure  $ABCD$  below, answer the following questions.



- Explain how you would create the star figure above using only rotations.
  - Explain how to create the star figure above using reflections and rotation.
  - Explain how to create the star figure above using only reflections. Explain your answer.
9. Given the rectangle  $ABCD$  below, answer the following questions.



- Can you transform the rectangle  $A'B'C'D'$  above using only rotations? Explain your answer.
- Describe a way to create the rectangle  $A'B'C'D'$ .
- Can you make the rectangle  $A'B'C'D'$  above using only reflections? Explain your answer.