# Lesson 14: Solving Equations Involving Linear Transformations of the Coordinate Plane

## Classwork

### Opening Exercise

Ahmad says the matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ applied to the point $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ will reflect the point to $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$. Randelle says that applying the matrix to the given point will produce a rotation of 180° about the origin. Who is correct? Explain your answer, and verify the result.

### Example 1

a. Describe a transformation not already discussed that results in an image point of $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$, and represent the transformation using a $2$ x $2$.

b. Determine whether any of the matrices listed represent linear transformations that can produce the image point $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$. Justify your answers by describing the transformations represented by the matrices.

i. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

ii. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$



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iii. $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

c. Suppose a linear transformation $L$ is represented by the matrix $\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$. Find a point $L\begin{bmatrix} x \\ y \end{bmatrix}$ so that $L\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

**Exercises 1–4**

1. Given the system of equations

$$2x + 5y = 4$$
$$3x - 8y = -25$$

a. Show how this system can be written as a statement about a linear transformation of the form $Lx = b$, with $x = \begin{bmatrix} x \\ y \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ -25 \end{bmatrix}$.

b. Determine whether $L$ has an inverse. If it does, compute $L^{-1}b$, and verify that the coordinates represent the solution to the system of equations.

2. The path of a piece of paper carried by the wind into a tree can be modeled with a linear transformation, where $L = \begin{bmatrix} 3 & -4 \\ 5 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$.

a. Write an equation that represents the linear transformation of the piece of paper.


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b. Solve the equation from part (a).

c. Use your solution to provide a reasonable interpretation of the path of the piece of paper under the transformation by the wind.

3. For each system of equations, write the system as a linear transformation represented by a matrix and apply inverse matrix operations to find the solution, or explain why this procedure cannot be performed.

a. $6x + 2y = 1$
$y = 3x + 1$

b. $4x - 6y = 10$
$2x - 3y = 1$

4. In a two-dimensional plane, $A$ represents a rotation of $30°$ counterclockwise about the origin, $B$ represents a reflection over the line $y = x$, and $C$ represents a rotation of $60°$ counterclockwise about the origin.

a. Write matrices $A$, $B$, and $C$.


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b. Transformations $A$, $B$, and $C$ are applied to point $\begin{bmatrix} x \\ y \end{bmatrix}$ successively and produce the image point $\begin{bmatrix} 1+2\sqrt{3} \\ 2-\sqrt{3} \end{bmatrix}$. Use inverse matrix operations to find $\begin{bmatrix} x \\ y \end{bmatrix}$.



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## Problem Set

1. In a two-dimensional plane, a transformation represented by $L = \begin{bmatrix} 1 & 5 \\ 2 & -4 \end{bmatrix}$ is applied to point $x$, resulting in an image point $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$. Find the location of the point before it was transformed.
   a. Write an equation to represent the linear transformation of point $x$.
   b. Solve the equation to find the coordinates of the pre-image point.

2. Find the location of the point $\begin{bmatrix} x \\ y \end{bmatrix}$ before it was transformed when given:
   a. The transformation $L = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and the resultant is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Verify your answer.
   b. The transformation $L = \begin{bmatrix} 4 & 7 \\ -1 & -2 \end{bmatrix}$ and the resultant is $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Verify your answer.
   c. The transformation $L = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$ and the resultant is $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Verify your answer.
   d. The transformation $L = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$ and the resultant is $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$. Verify your answer.
   e. The transformation $L = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ and the resultant is $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Verify your answer.

3. On a computer assembly line, a robot is placing a CPU onto a motherboard. The robot's arm is carried out by the transformation $L = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$.
   a. If the CPU is attached to the motherboard at point $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$, at what location does the robot pick up the CPU?
   b. If the CPU is attached to the motherboard at point $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$, at what location does the robot pick up the CPU?
   c. Find the transformation $L = \begin{bmatrix} -1 & c \\ b & 3 \end{bmatrix}$ that will place the CPU starting at $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ onto the motherboard at the location $\begin{bmatrix} -8 \\ 3 \end{bmatrix}$.

4. On a construction site, a crane is moving steel beams from a truck bed to workers. The crane is programed to perform the transformation $L = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$.
   a. If the workers are at location $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$, where does the truck driver need to unload the steel beams so that the crane can pick them up and bring them to the workers?
   b. If the workers move to another location $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$, where does the truck driver need to unload the steel beams so that the crane can pick them up and bring them to the workers?



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5. A video game soccer player is positioned at $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$, where he kicks the ball. The ball goes into the goal, which is at point $\begin{bmatrix} 10 \\ 0 \end{bmatrix}$. When the player moves to point $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and kicks the ball, he misses the goal. The ball lands at point $\begin{bmatrix} 10 \\ -1 \end{bmatrix}$. What is the program/transformation $L = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ that this video soccer player uses?

6. Tim bought $5$ shirts and $3$ pair of pants, and it cost him $\$250$. Scott bought $3$ shirts and $2$ pair of pants, and it cost him $\$160$. All the shirts have the same cost, and all the pants have the same cost.
   a. Write a system of linear equations to find the cost of the shirts and pants.
   b. Show how this system can be written as a statement about a linear transformation of the form $Lx = b$ with $x = \begin{bmatrix} S \\ P \end{bmatrix}$ and $b = \begin{bmatrix} 250 \\ 160 \end{bmatrix}$.
   c. Determine whether $L$ has an inverse. If it does, compute $L^{-1}b$, and verify your answer to the system of equations.

7. In a two-dimensional plane, $A$ represents a reflection over the $x$-axis, $B$ represents a reflection over the $y$-axis, and $C$ represents a reflection over the line $y = x$.
   a. Write matrices $A, B$, and $C$.
   b. Write an equation for each linear transformation, assuming that each one produces an image point of $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$.
   c. Use inverse matrix operations to find the pre-image point for each equation. Explain how your solutions make sense based on your understanding of the effect of each geometric transformation on the coordinates of the pre-image points.

8. A system of equations is shown:

$$2x + 5y + z = 3$$
$$4x + y - z = 5$$
$$3x + 2y + 4z = 1$$

   a. Represent this system as a linear transformation in three-dimensional space represented by a matrix equation in the form of $Lx = b$.
   b. What assumption(s) need to be made to solve the equation in part (a) for $x$.
   c. Use algebraic methods to solve the system.

9. Assume

$$L^{-1} = \frac{1}{78}\begin{bmatrix} -6 & 18 & 6 \\ 19 & -5 & -6 \\ -5 & -11 & 18 \end{bmatrix}$$

   Use inverse matrix operations to solve the equation from Problem 8, part (a) for $x$. Verify that your solution is the same as the one you found in Problem 8, part (c).


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