Lesson 21: Vectors and the Equation of a Line

Classwork

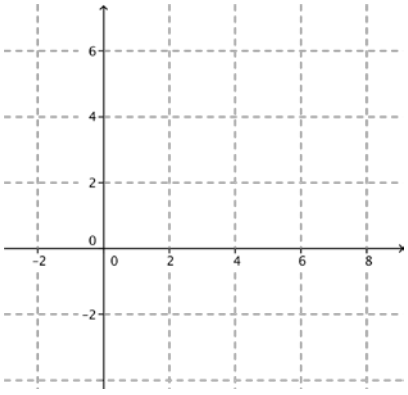
Opening Exercise

a. Find three different ways to write the equation that represents the line in the plane that passes through points $(1,2)$ and $(2,-1)$.

b. Graph the line through point $(1,1)$ with slope 2.

Exercises

1. Consider the line ℓ in the plane given by the equation $3x - 2y = 6$.
   a. Sketch a graph of line ℓ on the axes provided.



   b. Find a point on line ℓ and the slope of line ℓ.

c. Write a vector equation for line ℓ using the information you found in part (b).

d. Write parametric equations for line ℓ.

e. Verify algebraically that your parametric equations produce points on line ℓ.

2. Olivia wrote parametric equations $x(t) = 4 + 2t$ and $y(t) = 3 + 3t$. Are her equations correct? What did she do differently from you?

3. Convert the parametric equations $x(t) = 2 - 3t$ and $y(t) = 4 + t$ into slope-intercept form.

4. Find parametric equations to represent the line that passes through point $(4,2,9)$ and has direction vector $\vec{\mathbf{v}} = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$.

5. Find a vector form of the equation of the line given by the parametric equations

$$x(t) = 3t$$
$$y(t) = -4 - 2t$$
$$z(t) = 3 - t.$$

Lesson Summary

Lines in the plane and lines in space can be described by either a vector equation or a set of parametric equations.

- Let ℓ be a line in the plane that contains point (x_1, y_1) and has direction vector $\vec{\mathbf{v}} = \begin{bmatrix} a \\ b \end{bmatrix}$. If the slope of line ℓ is defined, then $m = \frac{b}{a}$.

A vector form of the equation that represents line ℓ is

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} t.$$

Parametric equations that represent line ℓ are

$$x(t) = x_1 + at$$
$$y(t) = y_1 + bt.$$

- Let ℓ be a line in space that contains point (x_1, y_1, z_1) and has direction vector $\vec{\mathbf{v}} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

A vector form of the equation that represents line ℓ is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} a \\ b \\ c \end{bmatrix} t.$$

Parametric equations that represent line ℓ are

$$x(t) = x_1 + at$$
$$y(t) = y_1 + bt$$
$$z(t) = z_1 + ct.$$

Problem Set

1. Find three points on the line in the plane with parametric equations $x(t) = 4 - 3t$ and $y(t) = 1 + \frac{1}{3}t$.

2. Find vector and parametric equations to represent the line in the plane with the given equation.
   a. $y = 3x - 4$
   b. $2x - 5y = 10$
   c. $y = -x$
   d. $y - 2 = 3(x + 1)$

3. Find vector and parametric equations to represent the following lines in the plane.
   a. the x-axis
   b. the y-axis

c. the horizontal line with equation $y = 4$
d. the vertical line with equation $x = -2$
e. the horizontal line with equation $y = k$, for a real number k
f. the vertical line with equation $x = h$, for a real number h

4. Find the point-slope form of the line in the plane with the given parametric equations.
   a. $x(t) = 2 - 4t, y(t) = 3 - 7t$
   b. $x(t) = 2 - \frac{2}{3}t, y(t) = 6 + t$
   c. $x(t) = 3 - t, y(t) = 3$
   d. $x(t) = t, y(t) = t$

5. Find vector and parametric equations for the line in the plane through point P in the direction of vector $\mathbf{v}$.
   a. $P = (1,5), \vec{\mathbf{v}} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$
   b. $P = (0,0), \vec{\mathbf{v}} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$
   c. $P = (-3,-1), \vec{\mathbf{v}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

6. Determine if the point A is on the line ℓ represented by the given parametric equations.
   a. $A = (3,1), x(t) = 1 + 2t$ and $y(t) = 3 - 2t$.
   b. $A = (0,0), x(t) = 3 + 6t$ and $y(t) = 2 + 4t$
   c. $A = (2,3), x(t) = 4 - 2t$ and $y(t) = 4 + t$
   d. $A = (2,5), x(t) = 12 + 2t$ and $y(t) = 15 + 2t$

7. Find three points on the line in space with parametric equations $x(t) = 4 + 2t$, $y(t) = 6 - t$, and $z(t) = t$.

8. Find vector and parametric equations to represent the following lines in space.
   a. the x-axis
   b. the y-axis
   c. the z-axis

9. Convert the equation given in vector form to a set of parametric equations for the line ℓ.
   a. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} t$
   b. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} t$

c. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ -3 \\ -8 \end{bmatrix} t$

10. Find vector and parametric equations for the line in space through point P in the direction of vector $\vec{\mathbf{v}}$.

a. $P = (1,4,3), \vec{\mathbf{v}} = \begin{bmatrix} 3 \\ 6 \\ -2 \end{bmatrix}$

b. $P = (2,2,2), \vec{\mathbf{v}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

c. $P = (0,0,0), \vec{\mathbf{v}} = \begin{bmatrix} 4 \\ 4 \\ -2 \end{bmatrix}$

11. Determine if the point A is on the line ℓ represented by the given parametric equations.

a. $A = (3,1,1), x(t) = 5 - t, y(t) = -5 + 3t$ and $z(t) = 9 - 4t$

b. $A = (1,0,2), x(t) = 7 - 2t, y(t) = 3 - t$ and $z(t) = 4 - t$

c. $A = (5,3,2), x(t) = 8 + t, y(t) = -t$ and $z(t) = -4 - 2t$