Name ____ Date ____________________

Lesson 1: Solutions to Polynomial Equations

Exit Ticket

1. Find the solutions of the equation $x^4 - x^2 - 12$. Show your work.

2. The number 1 is a zero of the polynomial $p(x) = x^3 - 3x^2 + 7x - 5$.

   a. Write $p(x)$ as a product of linear factors.

   b. What are the solutions to the equation $x^3 - 3x^2 + 7x - 5 = 0$?

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Lesson 2: Does Every Complex Number Have a Square Root?

Exit Ticket

1. Find the two square roots of $5 - 12i$.

2. Find the two square roots of $3 - 4i$.

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Lesson 3: Roots of Unity

Exit Ticket

1. What is a fourth root of unity? How many fourth roots of unity are there? Explain how you know.

2. Find the polar form of the fourth roots of unity.

3. Write $x^4 - 1$ as a product of linear factors, and explain how this expression supports your answers to Problems 1 and 2.

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Lesson 4: The Binomial Theorem

Exit Ticket

1. Evaluate the following expressions.
   a. $5!$

   b. $\frac{8!}{6!}$

   c. $C(7,3)$

2. Find the coefficients of the terms below in the expansion of $(u + v)^8$. Explain your reasoning.
   a. u^2v^6

   b. u^3v^5

   c. u^4v^4

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Lesson 5: The Binomial Theorem

Exit Ticket

The area and circumference of a circle of radius r are given by:

$$A(r) = \pi r^2$$

$$C(r) = 2\pi r$$

a. Show mathematically that the average rate of change of the area of the circle as the radius increases from r to $r + 0.01$ units is very close to the perimeter of the circle.

b. Explain why this makes sense geometrically.

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Lesson 6: Curves in the Complex Plane

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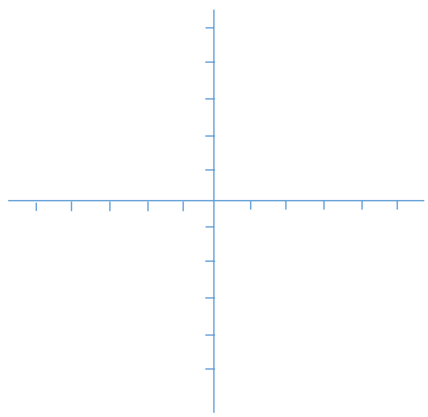
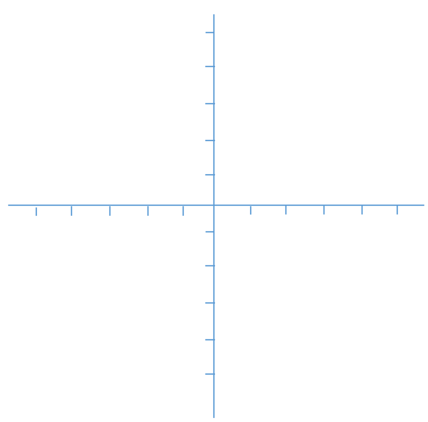
1. Write the real form of the complex equation $z = \cos(\theta) + 3i\sin(\theta)$. Sketch the graph of the equation.



2. Write the complex form of the equation $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Sketch the graph of the equation.



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Lesson 7: Curves from Geometry

Exit Ticket

Suppose that the foci of an ellipse are $F(-1,0)$ and $G(1,0)$ and that the point $P(x,y)$ satisfies the condition $PF+PG=4$.

a. Derive an equation of an ellipse with foci F and G that passes through P. Write your answer in standard form: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

b. Sketch the graph of the ellipse defined above.

c. Verify that the x-intercepts of the graph satisfy the condition $PF+PG=4$.

d. Verify that the y-intercepts of the graph satisfy the condition $PF+PG=4$.

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Lesson 8: Curves from Geometry

Exit Ticket

Let $F(-4,0)$ and $B(4,0)$ be the foci of a hyperbola. Let the points $P(x,y)$ on the hyperbola satisfy either $PF - PG = 4$ or $PG - PF = 4$. Derive an equation for this hyperbola, writing your answer in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

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Curve	Equation	Center	Asymptotes	Symmetry	Graph
Ellipse					
Hyperbola opening up/down					
Hyperbola opening left/right					

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Lesson 9: Volume and Cavalieri's Principle

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Explain how Cavalieri's principle can be used to find the volume of any solid.

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1.

a. Write $(1+i)^7 - (1-i)^7$ in the form $a + bi$ for some real numbers a and b.

b. Explain how Pascal's Triangle allows you to compute the coefficient of x^2y^3 when $(x-y)^5$ is expanded.

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2. Verify that the fundamental theorem of algebra holds for the fourth-degree polynomial p given by $p(z) = z^4 + 1$ by finding four zeros of the polynomial and writing the polynomial as a product of four linear terms. Be sure to make use of the polynomial identity given below.

$$x^4 - a^4 = (x - a)(x + a)(x - ai)(x + ai)$$

3. Consider the cubic polynomial p given $p(z) = z^3 - 8$.

a. Find a real number root to the polynomial.

b. Write $p(z)$ as a product of three linear terms.

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Consider the degree-eight polynomial q given by $q(z) = z^8 - 2^8$.

c. What is the largest possible number of distinct roots the polynomial q could possess? Briefly explain your answer.

d. Find all the solutions to $q(z) = 0$.

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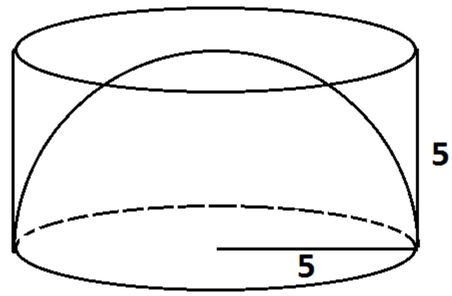
4.

a. A right circular cylinder of radius 5 cm and height 5 cm contains half a sphere of radius 5 cm as shown.



Use Cavalieri's Principle to explain why the volume inside this cylinder but outside the hemisphere is equivalent to the volume of a circular cone with base of radius 5 cm and height 5 cm.

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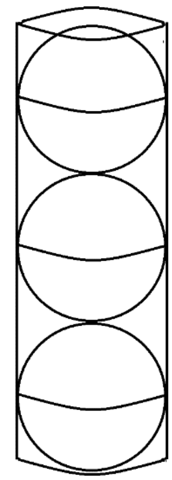
b. Three congruent solid balls are packaged in a cardboard cylindrical tube. The cylindrical space inside the tube has dimensions such that the three balls fit snugly inside that tube as shown.



Each ball is composed of material with density 15 grams per cubic centimeter. The space around the balls inside the cylinder is filled with aerated foam with a density of 0.1 grams per cubic centimeter.

i. Ignoring the cardboard of the tube, what is the average density of the inside contents of the tube?

ii. If the contents inside the tube, the three balls and the foam, weigh 150 grams to one decimal place, what is the weight of one ball in grams?

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5.

a. Consider the two points $F(-9,0)$ and $G(9,0)$ in the coordinate plane. What is the equation of the ellipse given as the set of all points P in the coordinate plane satisfying $FP + PG = 30$? Write the equation in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with a and b real numbers, and explain how you obtain your answer.

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b. Consider again the two points $F(-9, 0)$ and $G(9, 0)$ in the coordinate plane. The equation of the hyperbola defined by $|FP - PG| = k$ for some constant k is given by $\frac{x^2}{25} - \frac{y^2}{56} = 1$. What is the value of k?

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Lesson 10: The Structure of Rational Expressions

Exit Ticket

1. Payton says that rational expressions are not closed under addition, subtraction, multiplication, or division. His claim is shown below. Is he correct for each case? Justify your answers.

   a. $\frac{x}{2x+1} + \frac{x+1}{2x+1} = 1$, and 1 is a whole number not a rational expression.

   b. $\frac{3x-1}{2x+1} - \frac{3x-1}{2x+1} = 0$, and 0 is a whole number not a rational expression.

   c. $\frac{x-1}{x+1} \cdot \frac{x+1}{1} = x - 1$, and $x - 1$ is a whole number not a rational expression.

   d. $\frac{x-1}{x+1} \div \frac{1}{x+1} = x - 1$, and $x - 1$ is a whole number not a rational expression.

2. Simplify the following rational expressions by rewriting them with a single polynomial denominator.

   a. $\frac{3}{x-1} + \frac{2}{x}$

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b. $\frac{2}{x-2} - \frac{3}{x}$

c. $\frac{x+1}{x-1} \cdot \frac{x}{x-1}$

d. $\frac{x+2}{x-1} \div \frac{x-2}{x^2-1}$

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Lesson 11: Rational Functions

Exit Ticket

1. Identify whether the functions shown are rational:

   a. $f(x) = \frac{x}{x^2+1}$

   b. $f(x) = \frac{\sqrt{x}}{x^2+1}$

   c. $f(x) = \frac{x}{x^{2/3}+1}$

   d. $f(x) = \left(\frac{x}{x^2+1}\right)^2$

   e. $f(x) = \frac{\sqrt{2}x}{ex^2+\sqrt{\pi}}$

2. Anmol says $f(x) = \frac{x+1}{x^2-1}$ and $g(x) = \frac{1}{x-1}$ represent the same function. Is she correct? Justify your answer.

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Lesson 12: End Behavior of Rational Functions

Exit Ticket

Given $(x) = \frac{x+2}{x^2-1}$, find the following, and justify your findings.

a. The end-behavior model for the numerator.

b. The end-behavior model for the denominator.

c. The end-behavior model for $f(x)$.

d. What is the value of $f(x)$ as $x \to \infty$?

e. What is the value of $f(x)$ as $x \to -\infty$?

f. What is the value of $f(x)$ as $x \to 1$ from the positive and negative sides?

g. What is the value of $f(x)$ as $x \to -1$ from the positive and negative sides?

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Lesson 13: Horizontal and Vertical Asymptotes of Graphs of Rational Functions

Exit Ticket

Consider the function $(x) = \frac{-2x+5}{x^2-5x-6}$.

1. Looking at the structure of the function, what information can you gather about the graph of f?

2. State the domain of f.

3. Determine the end behavior of f.

4. State the equations of any vertical and horizontal asymptotes on the graph of $y = f(x)$.

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Lesson 14: Graphing Rational Functions

Exit Ticket

1. Sketch the graph of $y = \frac{x-3}{x^2+x-12}$. Label horizontal and vertical asymptotes, and identify any discontinuities, x-intercepts, and the y-intercept if they exist. Describe the end behavior of the function.

2. Does the graph of the function $f(x) = \frac{x^2-8x-9}{x+1}$ have a vertical asymptote or a point missing at $x = -1$? Explain your reasoning, and support your answer numerically.

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Lesson 15: Transforming Rational Functions

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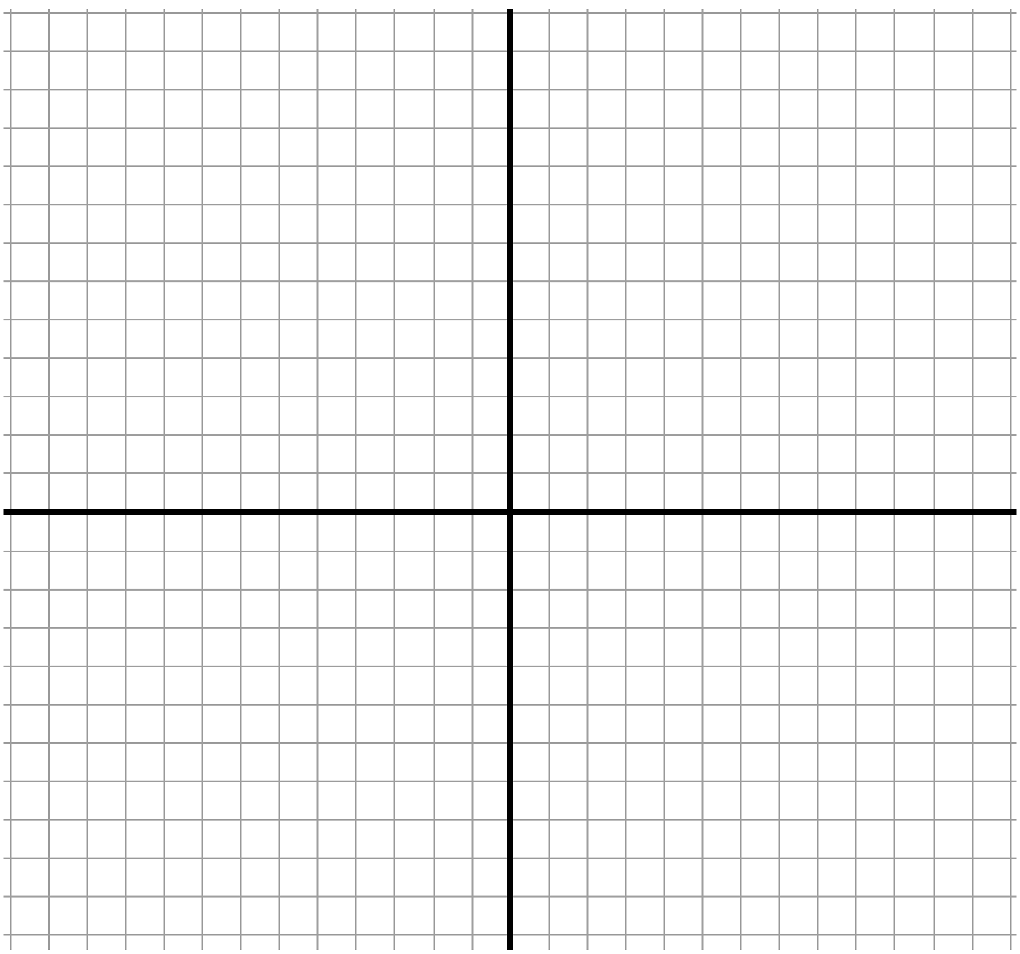
Sketch the graph of the function given below by using transformations of $y = \frac{1}{x^n}$. Explain which transformations you used and how you identified them.

$$y = \frac{3x - 7}{x - 3}$$



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Lesson 16: Function Composition

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1. Let $f(x) = x^2$ and $g(x) = 2x + 3$. Write an expression that represents each composition:
   a. $(g \circ f)x$

   b. $f\big(f(-2)\big)$

   c. $(f \circ g)\left(\frac{1}{x}\right)$

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2. A consumer advocacy company conducted a study to research the pricing of fruits and vegetables. They collected data on the size and price of produce items, including navel oranges. They found that, for a given chain of stores, the price of oranges was a function of the weight of the oranges, $p = f(w)$.

w weight in pounds	0.2	0.25	0.3	0.4	0.5	0.6	0.7
p price in dollars	0.26	0.32	0.39	0.52	0.65	0.78	0.91

The company also determined that the weight of the oranges measured was a function of the radius of the oranges, $w = g(r)$.

r radius in inches	1.5	1.65	1.7	1.9	2	2.1
w in pounds	0.38	0.42	0.43	0.48	0.5	0.53

a. How can the researcher use function composition to examine the relationship between the radius of an orange and its price? Use function notation to explain your response.

b. Use the table to evaluate $f\big(g(2)\big)$, and interpret this value in context.

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Lesson 17: Solving Problems by Function Composition

Exit Ticket

Timmy wants to install a wooden floor in a square room. The cost to install the floor is $24 per 4 square feet.

a. Write a function to find the area of the room as a function of its length.

b. Write a function for the cost to install the floor as a function of its area.

c. Write a function to find the total cost to install the floor.

d. Show how the function in part (c) is the result of a composition of two functions.

e. How much does it cost to install a wood floor in a square room with a side length of 10 feet?

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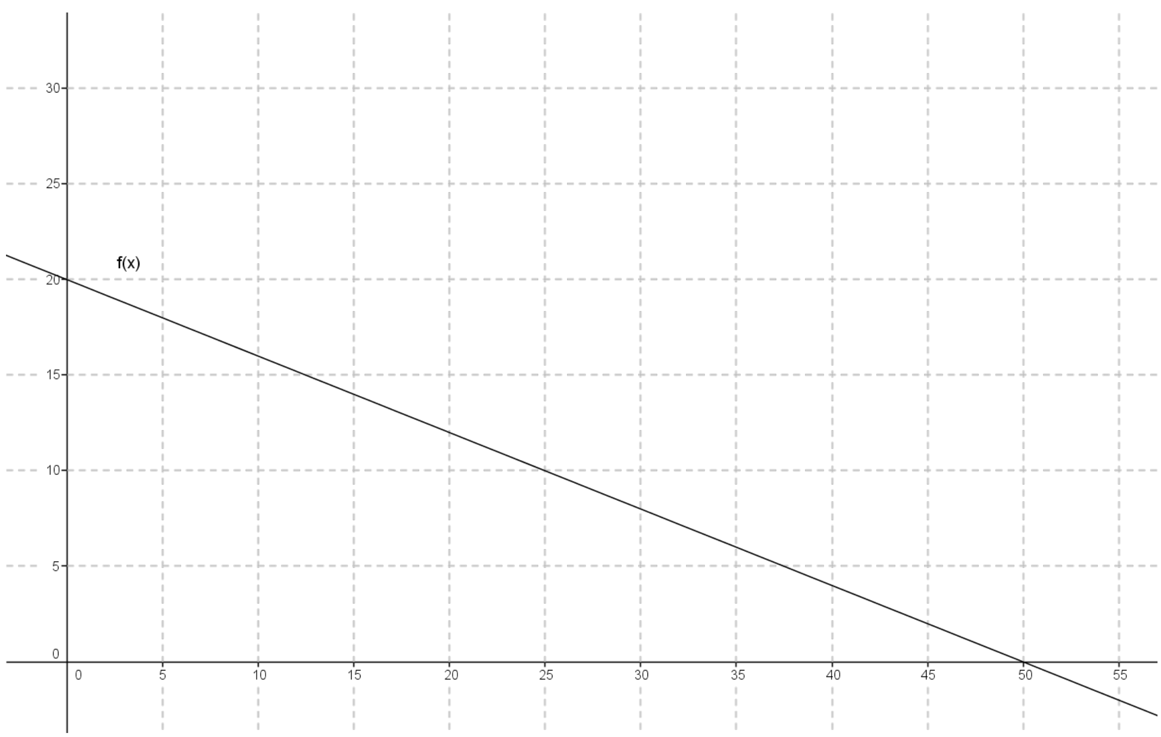
Lesson 18: Inverse Functions

Exit Ticket

The function f is described below in three different ways. For each way, express f^{-1} in the same style.

x	1	2	5	10	15	20
$f(x)$	19.6	19.2	18	16	14	12

$$f(x) = -\frac{2}{5}x + 20$$



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Lesson 19: Restricting the Domain

Exit Ticket

Let $f(x) = x^2 - 3x + 2$.

a. Give a restricted domain for f where it is invertible.

b. Find the inverse of f for the domain you gave in part (a).

c. State the domain and range of the function you found in part (b).

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d. Verify through function composition that the function you found in part (b) is the inverse of f.

e. Graph both functions on the domains specified.

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Lesson 20: Inverses of Logarithmic and Exponential Functions

Exit Ticket

1. Find the inverse of each function.
   a. $f(x) = \log_2(x) + 2$

   b. $g(x) = e^{x-4}$

   c. $h(x) = 3\log(2 + 3x)$

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2. Verify by composition that the given functions are inverses.

   a. $f(x) = 2 - \log(3y + 2)\,;\ g(x) = \frac{1}{3}(100 \cdot 10^{-x} - 2)$

   b. $f(x) = \ln(x) - \ln(x + 1)\,;\ \ g(x) = \frac{e^x}{1 - e^x}$

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Lesson 21: Logarithmic and Exponential Problem Solving

Exit Ticket

Darrin drank a latte with 205 mg of caffeine. Each hour, the caffeine in Darrin's body diminishes by about 8%.

a. Write a formula to model the amount of caffeine remaining in Darrin's system after each hour.

b. Write a formula to model the number of hours since Darrin drank his latte based on the amount of caffeine in Darrin's system.

c. Use your equation in part (b) to find how long it will take for the caffeine in Darrin's system to drop below 50 mg.

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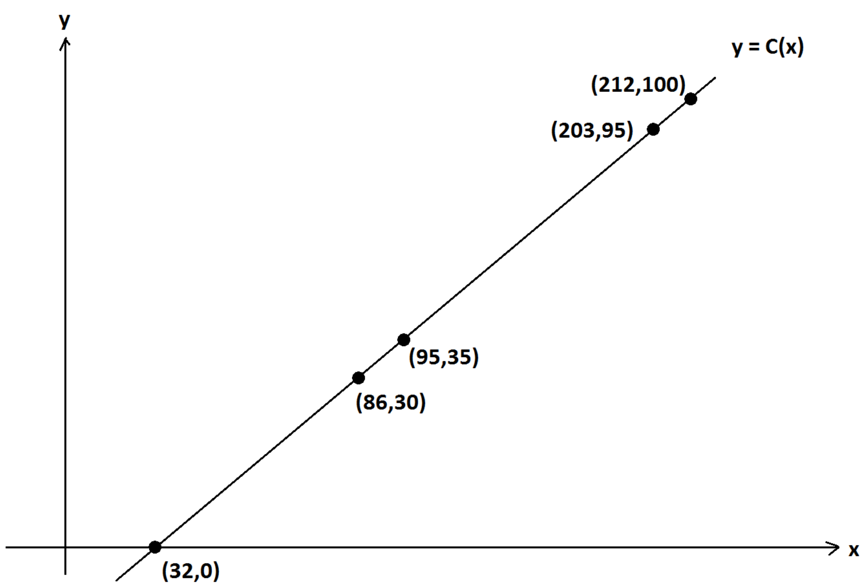
1. Let C be the function that assigns to a temperature given in degrees Fahrenheit its equivalent in degrees Celsius, and let K be the function that assigns to a temperature given in degrees Celsius its equivalent in degrees Kelvin.

   We have $C(x) = \frac{5}{9}(x - 32)$ and $K(x) = x + 273$.

   a. Write an expression for $K(C(x))$ and interpret its meaning in terms of temperatures.

   b. The following shows the graph of $y = C(x)$.

   According to the graph, what is the value of $C^{-1}(95)$?



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c. Show that $C^{-1}(x) = \frac{9}{5}x + 32$.

A weather balloon rises vertically directly above a station at the North Pole. Its height at time t minutes is $H(t) = 500 - \frac{500}{2^t}$ meters. A gauge on the balloon measures atmospheric temperature in degrees Celsius.

Also, let T be the function that assigns to a value y the temperature, measured in Kelvin, of the atmosphere y meters directly above the North Pole on the day and hour the weather balloon is launched. (Assume that the temperature profile of the atmosphere is stable during the balloon flight.)

d. At a certain time t minutes, $K^{-1}\left(T\big(H(t)\big)\right) = -20$. What is the readout on the temperature gauge on the balloon at this time?

e. Find, to one decimal place, the value of $H^{-1}(300) = -20$ and interpret its meaning.

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2. Let f and g be the functions defined by $f(x) = 10^{\frac{x+2}{3}}$ and $g(x) = \log\left(\frac{x^3}{100}\right)$ for all positive real numbers, x. (Here the logarithm is a base-ten logarithm.)

Verify by composition that f and g are inverse functions to each other.

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3. Water from a leaky faucet is dripping into a bucket. Its rate of flow is not steady, but it is always positive. The bucket is large enough to contain all the water that will flow from the faucet over any given hour.

The table below shows $V(t)$, the total amount of water in the bucket, measured in cubic centimeters, as a function of time t, measured in minutes, since the bucket was first placed under the faucet.

t (minutes)	**0**	**1**	**2**	**2.5**	**3.7**	**5**	**10**
V (cubic cm)	**0**	**10.2**	**25.1**	**32.2**	**40.4**	**63.2**	**69.2**

a. Explain why V is an invertible function.

b. Find $V^{-1}(63.2)$ and interpret its meaning in the context of this situation.

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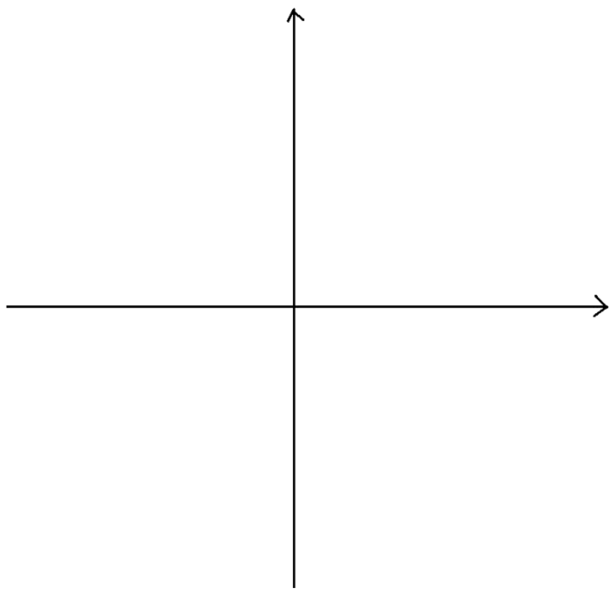
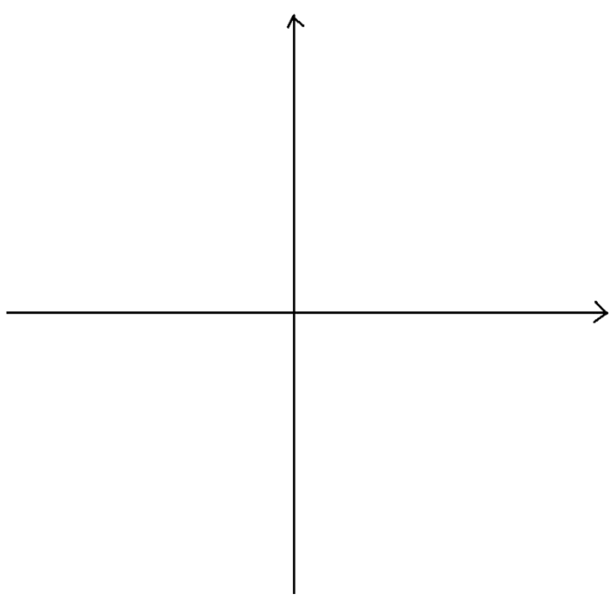
4.

a. Draw a sketch of the graph of $y = \frac{1}{x}$.



b. Sketch the graph of $y = \frac{x}{x-1}$, being sure to indicate its vertical and horizontal asymptotes.



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Let f be the function defined by $f(x) = \frac{x}{x-1}$ for all real values x different from 1.

c. Find $f(f(x))$ for x, a real number different from 1. What can you conclude about $f^{-1}(x)$?

5. Let f be the function given by $f(x) = x^2 + 3$.

a. Explain why f is not an invertible function on the domain of all real numbers.

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b. Describe a set S of real numbers such that if we restrict the domain of f to S, the function f has an inverse function. Be sure to explain why f has an inverse for your chosen set S.

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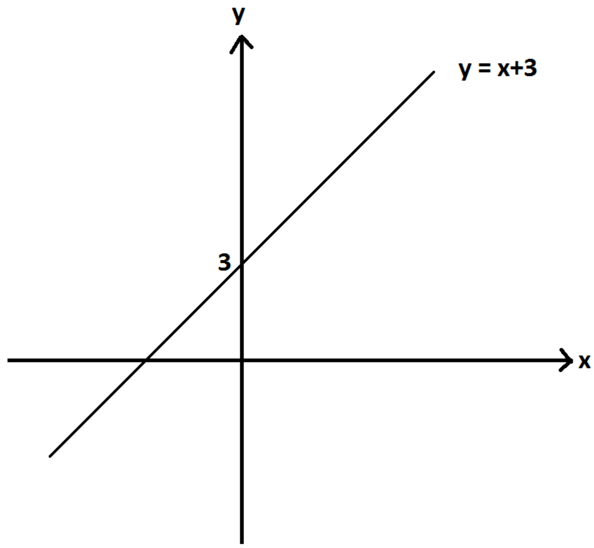
6. The graph of $y = x + 3$ is shown below.



Consider the rational function h given by $h(x) = \frac{x^2-x-12}{x-4}$.

Simon argues that the graph of $y = h(x)$ is identical to the graph of $y = x + 3$. Is Simon correct? If so, how does one reach this conclusion? If not, what is the correct graph of $y = h(x)$? Explain your reasoning throughout.

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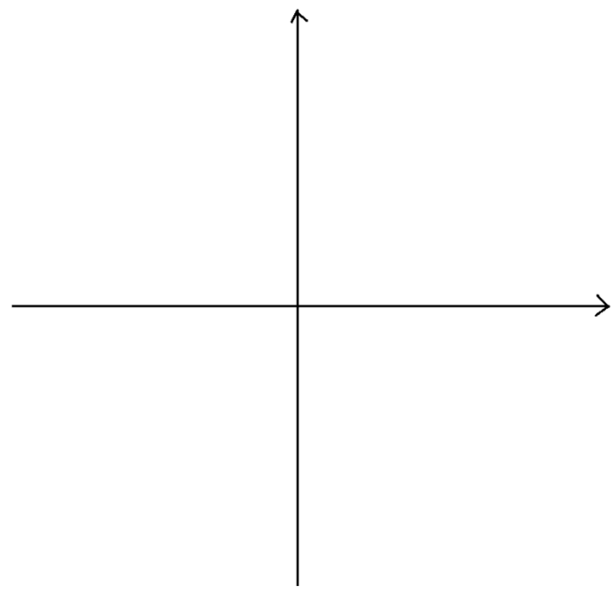
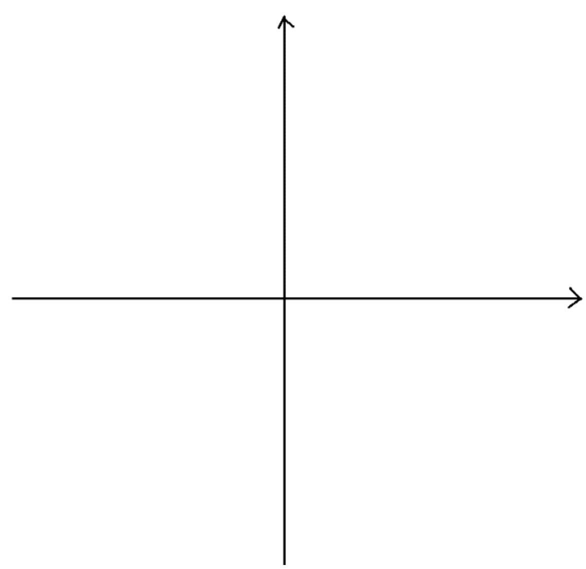
7. Let f be the function given by $f(x) = 2^x$ for all real values x, and let g be the function given by $g(x) = log_2(x)$ for positive real values x.

   a. Sketch a graph of $y = f(g(x))$. Describe any restrictions on the domain and range of the functions and the composite functions.



   b. Sketch a graph of $y = g(f(x))$. Describe any restrictions on the domain and range of the functions and the composite functions.



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8. Let f be the rational function given by $f(x) = \frac{x+2}{x-1}$ and g the rational function given by $g(x) = \frac{x-2}{x+1}$.

   a. Write $f(x) \div g(x)$ as a rational expression.

   b. Write $f(x) + g(x)$ as a rational expression.

   c. Write $f(x) - g(x)$ as a rational expression.

   d. Write $\frac{2f(x)}{f(x)+g(x)}$ as a rational expression.

   e. Ronaldo says that f is the inverse function to g. Is he correct? How do you know?

   f. Daphne says that the graph of f and the graph of g each have the same horizontal line as a horizontal asymptote. Is she correct? How do you know?

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Let $r(x) = f(x) \cdot g(x)$, and consider the graph of $y = r(x)$.

g. What are the x-intercepts of the graph of $y = r(x)$?

h. What is the y-intercept of $y = r(x)$?

i. At which x-values is $r(x)$ undefined?

j. Does the graph of $y = r(x)$ have a horizontal asymptote? Explain your reasoning.

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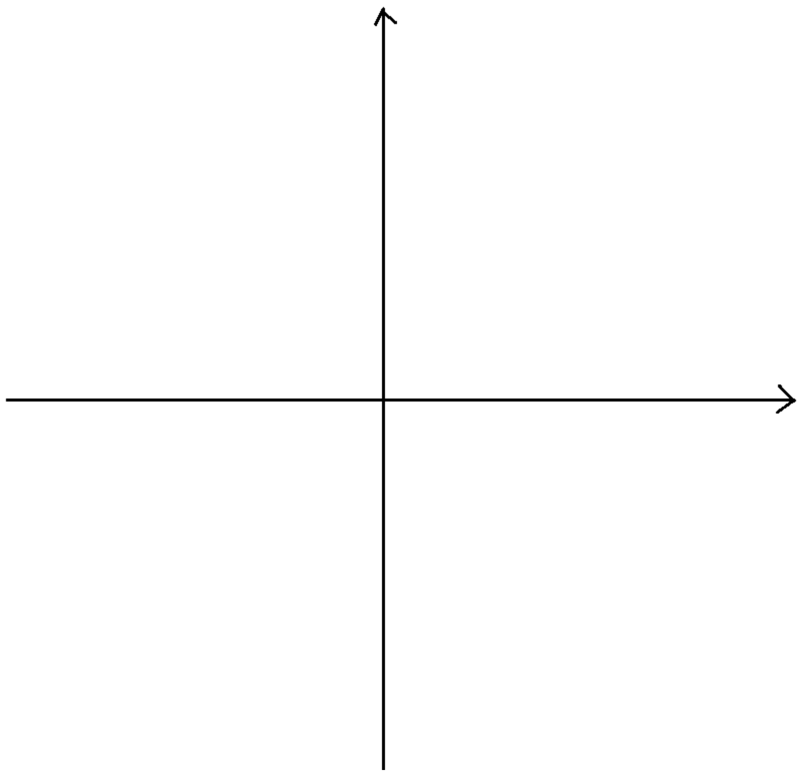
k. Give a sketch of the graph of $y = r(x)$ which shows the broad features you identified in parts (g)–(j).



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9. An algae growth in an aquarium triples in mass every two days. The mass of algae was 2.5 grams on June 21, considered *day zero*, and the following table shows the mass of the algae on later days.

d (day number)	**0**	**2**	**3**	**4**	**8**	**10**
mass (grams)	**2.5**	**7.5**	**13.0**	**22.5**	**202.5**	**607.5**

Let $m(d)$ represent the mass of the algae, in grams, on day d. Thus, we are regarding m as a function of time given in units of days. Our time measurements need not remain whole numbers. (We can work with fractions of days too, for example.)

a. Explain why m is an invertible function of time.

b. According to the table, what is the value of $m^{-1}(202.5)$? Interpret its meaning in the context of this situation.

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c. Find a formula for the inverse function m, and use your formula to find the value of $m^{-1}(400)$ to one decimal place.

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