Lesson 10: The Structure of Rational Expressions

Classwork

Opening Exercise

1. Add the fractions: $\frac{3}{5}+\frac{2}{7}$.

2. Subtract the fractions: $\frac{5}{2}-\frac{4}{3}$.

3. Add the expressions: $\frac{3}{x}+\frac{x}{5}$.

4. Subtract the expressions: $\frac{x}{x+2}-\frac{3}{x+1}$.

Exercises

1. Construct an argument that shows that the set of rational numbers is closed under addition. That is, if x and y are rational numbers and $w = x + y$, prove that w must also be a rational number.

2. How could you modify your argument to show that the set of rational numbers is also closed under subtraction? Discuss your response with another student.

3. Multiply the fractions: $\frac{2}{5} \cdot \frac{3}{4}$.

4. Divide the fractions: $\frac{2}{5} \div \frac{3}{4}$.

5. Multiply the expressions: $\frac{x+1}{x+2} \cdot \frac{3x}{x-4}$.

6. Divide the expressions: $\frac{x+1}{x+2} \div \frac{3x}{x-4}$.

7. Construct an argument that shows that the set of rational numbers is closed under division. That is, if x and y are rational numbers (with y nonzero) and $w = \frac{x}{y}$, prove that w must also be a rational number.

8. How could you modify your argument to show that the set of rational expressions is also closed under division by a nonzero rational expression? Discuss your response with another student.

Problem Set

1. Given $\frac{x+1}{x-2}$ and $\frac{x-1}{x^2-4}$ show that performing the following operations results in another rational expression.
   a. Addition.
   b. Subtraction.
   c. Multiplication.
   d. Division.

2. Find two rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ that produce the result $\frac{x-1}{x^2}$ when using the following operations. Answers for each type of operation may vary. Justify your answers.
   a. Addition.
   b. Subtraction.
   c. Multiplication.
   d. Division.

3. Find two rational expressions $\frac{a}{b}$ and $\frac{c}{d}$ that produce the result $\frac{2x+2}{x^2-x}$ when using the following operations. Answers for each type of operation may vary. Justify your answers.
   a. Addition.
   b. Subtraction.
   c. Multiplication.
   d. Division.

4. Consider the rational expressions A, B and their quotient, $\frac{A}{B}$, where B is not equal to zero.
   a. For some rational expression C, does $\frac{AC}{BC} = \frac{A}{B}$?
   b. Let $A = \frac{x}{y} + \frac{1}{x}$ and $B = \frac{y}{x} + \frac{1}{y}$. What is the least common denominator of every term of each expression?
   c. Find AC, BC where C is equal to your result in part (b). Then find $\frac{AC}{BC}$. Simplify your answer.
   d. Express each rational expression A, B as a single rational term; that is, as a division between two polynomials.
   e. Write $\frac{A}{B}$ as a multiplication problem.
   f. Use your answers to parts (d) and (e) to simplify $\frac{A}{B}$.
   g. Summarize your findings. Which method do you prefer using to simplify rational expressions?

5. Simplify the following rational expressions.
   a. $\dfrac{\frac{1}{y}-\frac{1}{x}}{\frac{x}{y}-\frac{y}{x}}$.

b. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$.

c. $\dfrac{\frac{1}{x^4}-\frac{1}{y^2}}{\frac{1}{x^4}+\frac{2}{x^2y}+\frac{1}{y^2}}$.

d. $\dfrac{\frac{1}{x-1}-\frac{1}{x}}{\frac{1}{x-1}+\frac{1}{x}}$.

6. Find A and B that make the equation true. Verify your results.

a. $\frac{A}{x+1}+\frac{B}{x-1}=\frac{2}{x^2-1}$.

b. $\frac{A}{x+3}+\frac{B}{x+2}=\frac{2x-1}{x^2+5x+6}$.

7. Find A, B, and C that make the equation true. Verify your result.

$\frac{Ax+B}{x^2+1}+\frac{C}{x+2}=\frac{x-1}{(x^2+1)(x+2)}$.