Lesson 16: Function Composition

Classwork

Example 1

Consider the tables from the opening scenario.

Depth of Free Diver During Descent									
s seconds of descent	20	40	60	80	100	120	140	160	180
d depth in meters of diver	15	32	44	65	79	90	106	120	133

Atmospheric Pressure and Ocean Depth									
d depth in meters of diver	10	20	30	40	50	60	70	80	90
p pressure in atm on diver	2	3	4	5	6	7	8	9	10

a. Do the tables appear to represent functions? If so, define the function represented in each table using a verbal description.

b. What are the domain and range of the functions?

c. Let's define the function in the first table as $d = f(s)$ and the function in the second table as $p = g(d)$. Use function notation to represent each output, and use the appropriate table to find its value:

i. depth of the diver at 80 seconds

ii. pressure of the diver at a depth of 60 meters

d. Explain how we could determine the pressure applied to a diver after 120 seconds of descending.

e. Use function notation to represent part (d), and use the tables to evaluate the function.

f. Describe the output from part (e) in context.

Example 2

Consider these functions:

f: Name→ Calendar Date

Assign each enrolled student to his or her birthday.

g: Name→ Name

Assign each person to his or her biological father.

Describe the action of each composite function. Determine which composite functions make sense.

a. $g \circ f$

b. $f \circ f$

c. $f \circ g$

d. $f \circ g \circ g$

Exercises 1–2

1. Let $f(x) = x^2$ and $g(x) = x + 5$. Write an expression that represents each composition:
   a. $(f \circ g)x$

   b. $g\big(f(4)\big)$

   c. $(f \circ g)(\sqrt{x+5})$

2. Suppose a sports medicine specialist is investigating the atmospheric pressure placed on competitive free divers during their descent. The following table shows the depth, d, in meters of a free diver s seconds into his descent. The depth of the diver is a function of the number of seconds the free diver has descended, $d = f(s)$.

s seconds	10	35	55	70	95	115	138	160	175
d depth in meters	8.1	28	45	55	76.0	91.5	110	130	145

The pressure, in atmospheres, felt on a free diver, d, is a function of his or her depth, $p = g(d)$.

d meters	25	35	55	75	95	115	135	155	175
p atm	2.4	3.5	5.5	7.6	9.6	11.5	13.7	15.5	17.6

a. How can the researcher use function composition to examine the relationship between the time a diver spends descending and the pressure he or she experiences? Use function notation to explain your response.

b. Explain the meaning of $g(f(0))$ in context.

c. Use the charts to approximate these values, if possible. Explain your answers in context.

i. $g(f(70))$

ii. $g(f(160))$

Problem Set

1. Determine whether each rule described represents a function. If the rule represents a function, write the rule using function notation, and describe the domain and range.
   a. Assign to each person his or her age in years.
   b. Assign to each person his or her height in centimeters.
   c. Assign to each piece of merchandise in a store a bar code.
   d. Assign each deli customer a number ticket.
   e. Assign a woman to her child.
   f. Assign to each number its first digit.
   g. Assign each person to his or her biological mother.

2. Let M: people → people
   Assign to each person his or her biological mother.
   F: people → people
   Assign to each person his or her biological father.
   L: people → people
   Assign to each person the first letter of his or her name.
   A: people → people
   Assign to each person his or her age in years.

   Which of the following compositions makes sense? For those that do, describe what the composite function is doing.
   a. $M \circ F$
   b. $L \circ M$
   c. $M \circ L$
   d. $A \circ M$
   e. $A \circ L$
   f. $F \circ M \circ A$
   g. $L \circ M \circ F$
   h. $A \circ M \circ M$

3. Let $f(x) = x^2 - x,\ g(x) = 1 - x$.
   a. $f \circ g$
   b. $g \circ f$
   c. $g \circ g$
   d. $f \circ f$
   e. $f(g(2))$
   f. $g(f(-1))$

4. Let $f(x) = x^2,\ g(x) = x + 3$.
   a. $g\big(f(5)\big)$
   b. $f\big(g(5)\big)$
   c. $f\big(g(x)\big)$
   d. $g\big(f(x)\big)$
   e. $g\left(f\left(\sqrt{x+3}\right)\right)$

5. Let $f(x) = x^3,\ g(x) = \sqrt[3]{x}$.
   a. $f \circ g$
   b. $g \circ f$
   c. $f\big(g(8)\big)$
   d. $g\big(f(2)\big)$
   e. $f\big(g(-8)\big)$
   f. $g\big(f(-2)\big)$

6. Let $f(x) = x^2,\ g(x) = \sqrt{x} + 3$.
   a. Show that $\big(f(x+3)\big) = |x+3| + 3$.
   b. Does $f(x) = |x+3| + 3 = (x) = |x| + 6$? Graph them on the same coordinate plane.

7. Given the chart below, find the following:

	-6	0	2	4
$f(x)$	4	-6	0	2
$g(x)$	2	4	-6	0
$h(x)$	0	2	4	-6
$k(x)$	1	4	0	3

   a. $f\big(g(0)\big)$
   b. $g\big(k(2)\big)$
   c. $k\big(g(-6)\big)$

d. $g(h(4))$
e. $g(k(4))$
f. $f \circ g \circ h(2)$
g. $f \circ f \circ f(0)$
h. $f \circ g \circ h \circ g(2)$

8. Suppose the strep throat virus is spreading in a community. The following table shows the number of people, n, that have the virus d days after the initial outbreak. The number of people who have the virus is a function of the number of days, $n = f(d)$.

d days	0	1	4	8	12	16	20
$n = f(d)$ number of people infected	2	4	14	32	64	50	30

There is only one pharmacy in the community. As the number of people who have the virus increases, the number of boxes of cough drops, b, sold also increases. The number of boxes of cough drops sold on a given day is a function of the number of people who have the virus, $b = g(n)$, on that day.

n number of people infected	0	2	4	9	14	20	28	32	44	48	50	60	64
$b = g(n)$ number of boxes of cough drops sold	1	5	14	16	22	30	42	58	74	86	102	124	136

a. Find $g(f(1))$, and state the meaning of the value in the context of the strep throat epidemic. Include units in your answer.

b. Fill the chart below using the fact that $b = g(f(d))$.

d (days)	0	1	4	8	12	16	20
b number of boxes of cough drops sold							

c. For each of the following expressions, interpret its meaning in the context of the problem, and if possible, give an approximation of its value.

i. $g(f(4))$
ii. $g(f(16))$
iii. $f(g(9))$