Lesson 19: Restricting the Domain

Classwork

Opening Exercise

The function f with domain $\{1,2,3,4,5\}$ is shown in the table below.

x	$f(x)$
1	7
2	3
3	1
4	9
5	5

a. What is $f(1)$? Explain how you know.

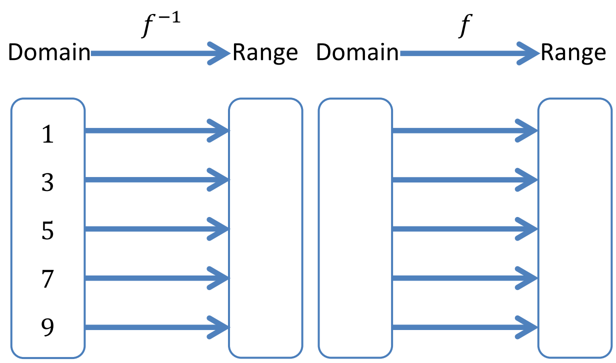
b. What is $f^{-1}(1)$? Explain how you know.

c. What is the domain of f^{-1}? Explain how you know.

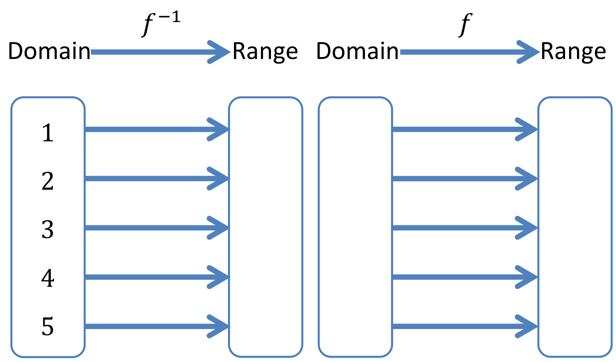
d. Construct a table for the function f^{-1}, the inverse of f.

Exercises 1–9

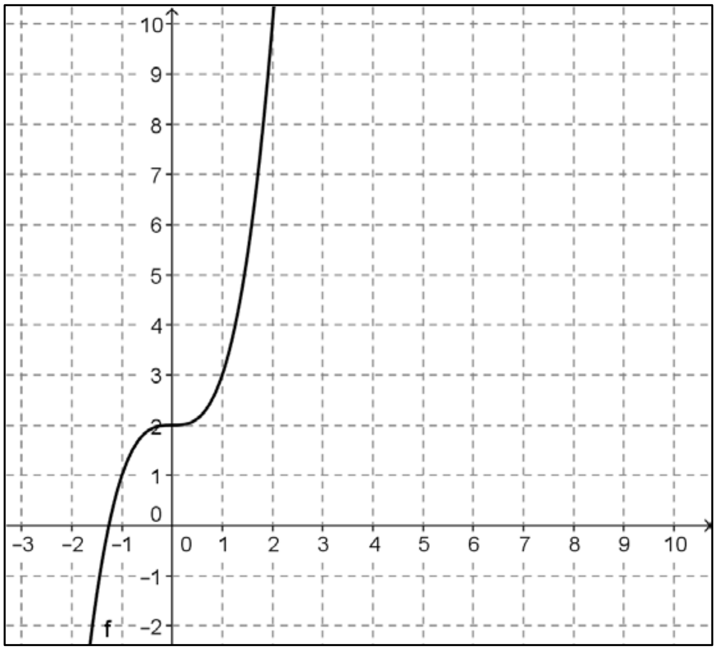
1. Complete the mapping diagram to show that $f\left(f^{-1}(x)\right) = x$.



2. Complete the mapping diagram to show that $f^{-1}\left(f(x)\right) = x$.

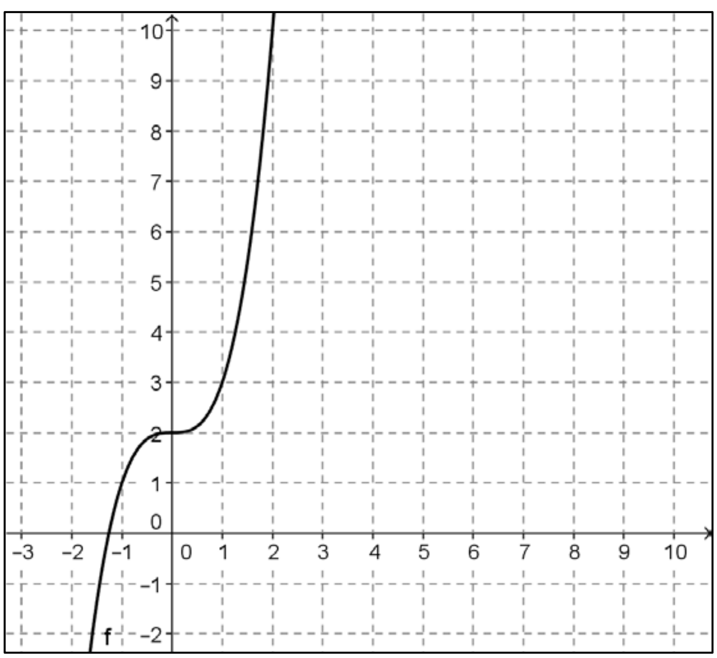


3. The graph of f is shown below.



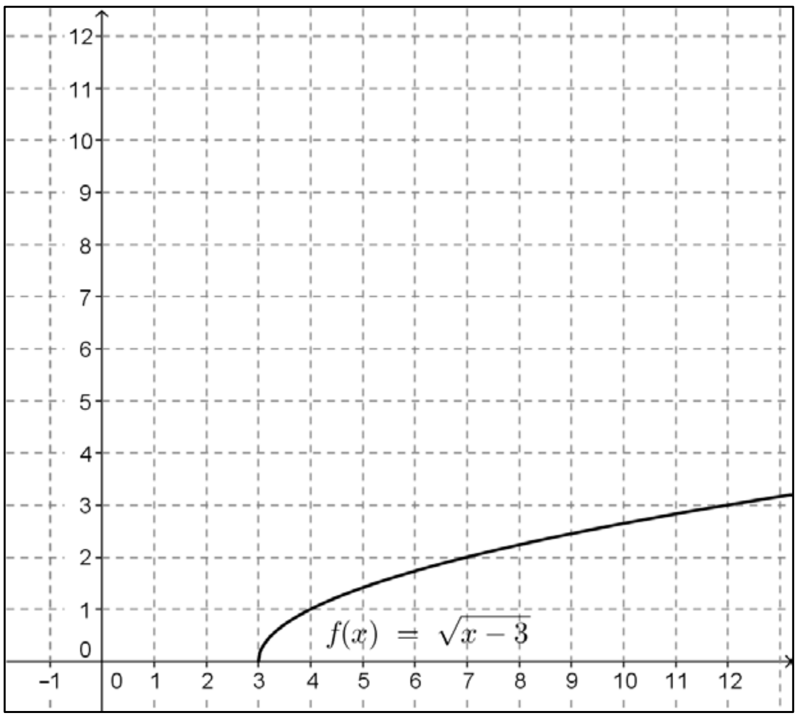
a. Select several ordered pairs on the graph of f, and use those to construct a graph of f^{-1}.

b. Draw the line $y = x$, and use it to construct the graph of f^{-1} below.



c. The algebraic function for f is given by $f(x) = x^3 + 2$. Is the formula for $f^{-1}(x) = \sqrt[3]{x} - 2$? Explain why or why not.

4. The graph of $f(x) = \sqrt{x - 3}$ is shown below. Construct the graph of f^{-1}.

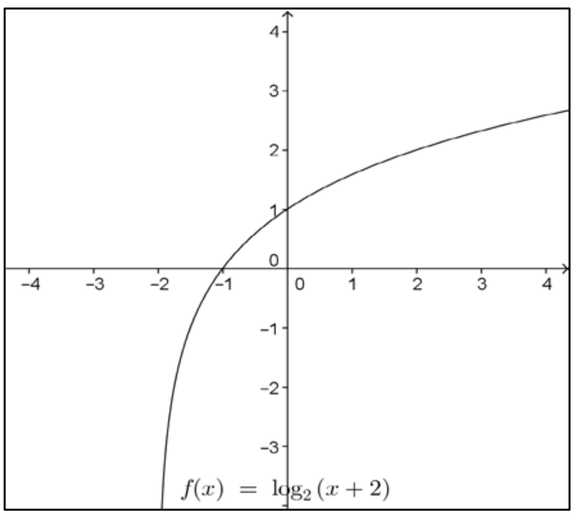


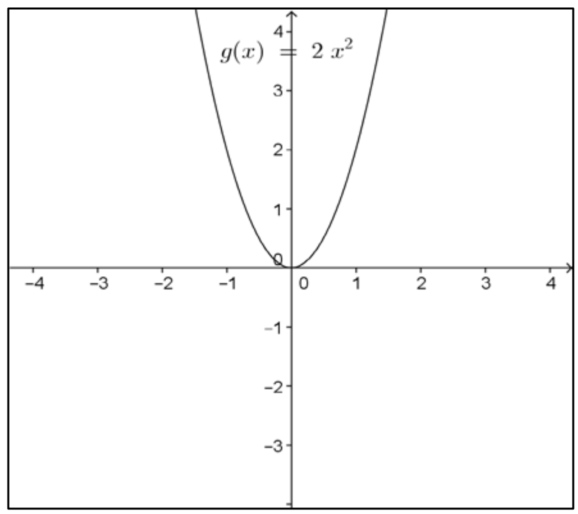
5. Morgan used the procedures learned in Lesson 18 to define $f^{-1}(x) = x^2 + 3$. How does the graph of this function compare to the one you made in Exercise 5?

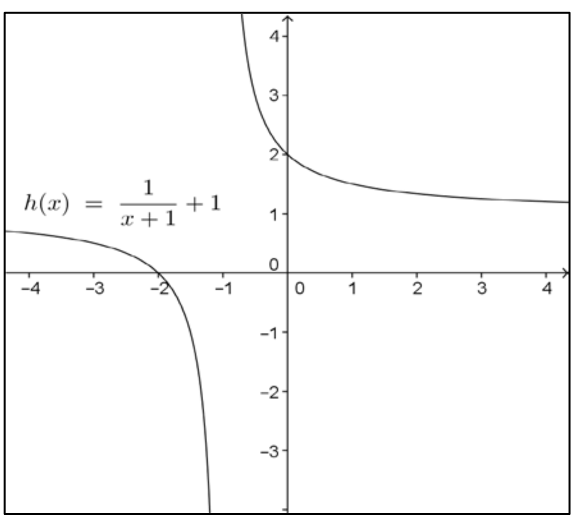
6. Construct the inverse of the function f given by the table below. Is the inverse a function? Explain your reasoning.

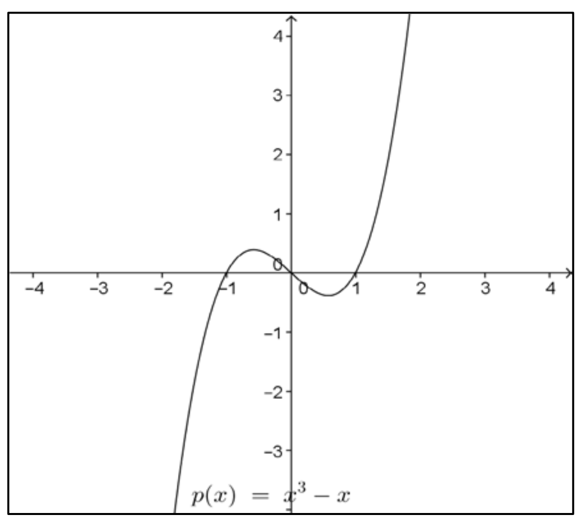
x	-3	-2	-1	0	1	2	3
$f(x)$	4	-1	-4	-5	-4	-1	4

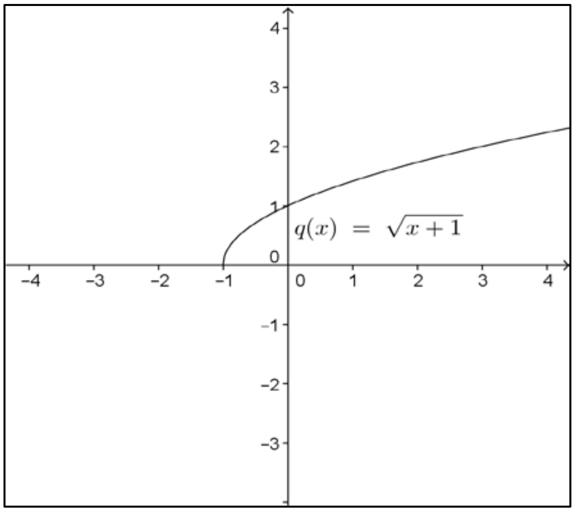
7. The graphs of several functions are shown below. Which ones are invertible? Explain your reasoning.

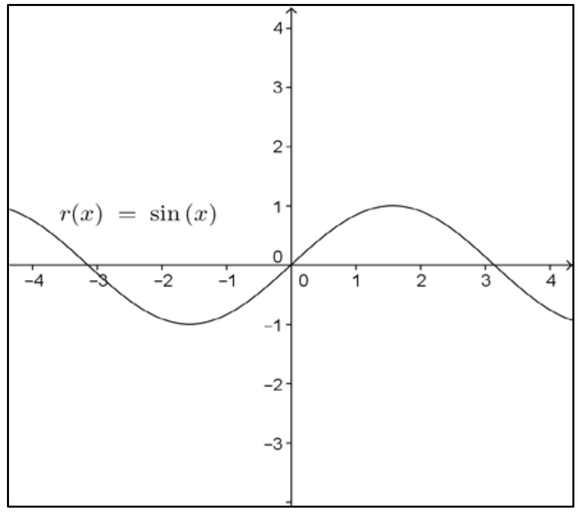












8. Given the function $f(x) = x^2 - 4$.

   a. Select a suitable domain for f that will make it an invertible function. State the range of f.

   b. Write a formula for f^{-1}. State the domain and range of f^{-1}.

c. Verify graphically that f, with the domain you selected, and f^{-1} are indeed inverses.

d. Verify that f and f^{-1} are indeed inverses by showing that $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$.

9. Three pairs of functions are given below. For which pairs, are f and g inverses of each other? Show work to support your reasoning. If a domain is not specified, assume it is the set of real numbers.

   a. $f(x) = \frac{x}{x+1}, x \neq -1$ and $g(x) = \frac{-x}{x-1}, x \neq 1$

   b. $f(x) = \sqrt{x} - 1, x \geq 0$ and $g(x) = (x+1)^2$

   c. $f(x) = -0.75x + 1$ and $g(x) = -\frac{4}{3}x - \frac{4}{3}$

Lesson Summary

COMPOSITION OF A FUNCTION AND ITS INVERSE: To verify that two functions are inverses, show that $f(g(x)) = x$ and $g(f(x)) = x$.

Invertible Function: The domain of a function f can be restricted to make it invertible.
A function is said to be invertible if its inverse is also a function.

Problem Set

1. Let f be the function that assigns to each student in your class his or her biological mother.
   a. In order for f to have an inverse, what condition must be true about the students in your class?
   b. If we enlarged the domain to include all students in your school, would this larger domain function have an inverse? Explain.

2. Consider a linear function of the form $f(x) = mx + b$, where m and b are real numbers, and $m \neq 0$..
   a. Explain why linear functions of this form always have an inverse this is also a function.
   b. State the general form of a line that does not have an inverse.
   c. What kind of function is the inverse of an invertible linear function (e.g., linear, quadratic, exponential, logarithmic, rational, etc.)?
   d. Find the inverse of a linear function of the form $f(x) = mx + b$, where m and b are real numbers, and $m \neq 0$.

3. Consider a quadratic function of the form $f(x) = b\left(\frac{x-h}{a}\right)^2 + k$ for real numbers a, b, h, k, and $a, b \neq 0$.
   a. Explain why quadratic functions never have an inverse without restricting the domain.
   b. What are the coordinates of the vertex of the graph of f?
   c. State the possible domains you can restrict f on so that it will have an inverse.
   d. What kind of function is the inverse of a quadratic function on an appropriate domain?
   e. Find f^{-1} for each of the domains you gave in part (c).

4. Show that $f(x) = mx + b$ for real numbers m and b with $m \neq 0$ has an inverse that is also a function.

5. Explain why $f(x) = a(x - h)^2 + k$ for real numbers a, h, and k with $a \neq 0$ does not have an inverse that is a function. Support your answer in at least two different ways (numerically, algebraically, or graphically).

Extension

6. Consider the function $f(x) = \sin(x)$.
   a. Graph $y = f(x)$ on the domain $[-2\pi, 2\pi]$.
   b. If we require a restricted domain on f to be continuous and cover the entirety of the range of f, how many possible choices for a domain are there in your graph from part (a)? What are they?
   c. Make a decision on which restricted domain you listed in part (b) makes the most sense to choose. Explain your decision.
   d. Use a calculator to evaluate $\sin^{-1}(0.75)$ to three decimal places. How can you use your answer to find other values ψ such that $\sin(\psi) = 1$? Verify that your technique works by checking it against your graph in part (a).