Lesson 1: Special Triangles and the Unit Circle

Classwork

Example 1

Find the following values for the rotation $\theta = \frac{\pi}{3}$ around the carousel. Create a sketch of the situation to help you. Interpret what each value means in terms of the position of the rider.

a. $\sin(\theta)$

b. $\cos(\theta)$

c. $\tan(\theta)$

Exercise 1

Assume that the carousel is being safety tested, and a safety mannequin is the rider. The ride is being stopped at different rotation values so technicians can check the carousel's parts. Find the sine, cosine, and tangent for each rotation indicated, and explain how these values relate to the position of the mannequin when the carousel stops at these rotation values. Use your carousel models to help you determine the values, and sketch your model in the space provided.

a. $\theta = \frac{\pi}{4}$

b. $\theta = \frac{\pi}{6}$

Example 2

Use your understanding of the unit circle and trigonometric functions to find the values requested.

a. $\sin\left(-\frac{\pi}{3}\right)$

b. $\tan\left(\frac{5\pi}{4}\right)$

Exercise 2

Use your understanding of the unit circle to determine the values of the functions shown.

a. $\sin\left(\frac{11\pi}{6}\right)$

b. $\cos\left(\frac{3\pi}{4}\right)$

c. $\tan(-\pi)$

Problem Set

1. Complete the chart below.

θ	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin(\theta)$			
$\cos(\theta)$			
$\tan(\theta)$			

2. Evaluate the following trigonometric expressions, and explain how you used the unit circle to determine your answer.
   a. $\cos\left(\pi + \frac{\pi}{3}\right)$
   b. $\sin\left(\pi - \frac{\pi}{4}\right)$
   c. $\sin\left(2\pi - \frac{\pi}{6}\right)$
   d. $\cos\left(\pi + \frac{\pi}{6}\right)$
   e. $\cos\left(\pi - \frac{\pi}{4}\right)$
   f. $\cos\left(2\pi - \frac{\pi}{3}\right)$
   g. $\tan\left(\pi + \frac{\pi}{4}\right)$
   h. $\tan\left(\pi - \frac{\pi}{6}\right)$
   i. $\tan\left(2\pi - \frac{\pi}{3}\right)$

3. Rewrite the following trigonometric expressions in an equivalent form using $\pi + \theta$, $\pi - \theta$, or $2\pi - \theta$ and evaluate.
   a. $\cos\left(\frac{\pi}{3}\right)$
   b. $\cos\left(\frac{-\pi}{4}\right)$
   c. $\sin\left(\frac{\pi}{6}\right)$
   d. $\sin\left(\frac{4\pi}{3}\right)$
   e. $\tan\left(\frac{-\pi}{6}\right)$
   f. $\tan\left(\frac{-5\pi}{6}\right)$

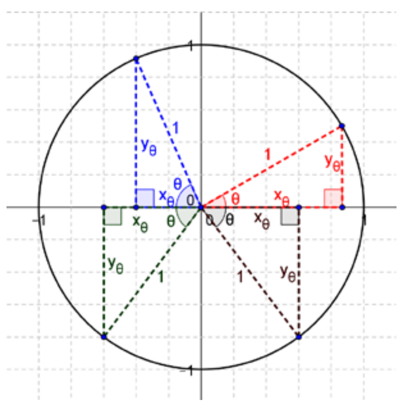
4. Identify the quadrant of the plane that contains the terminal ray of a rotation by θ if θ satisfies the given conditions.
   a. $\sin(\theta) > 0$ and $\cos(\theta) > 0$
   b. $\sin(\theta) < 0$ and $\cos(\theta) < 0$
   c. $\sin(\theta) < 0$ and $\tan(\theta) > 0$
   d. $\tan(\theta) > 0$ and $\sin(\theta) > 0$
   e. $\tan(\theta) < 0$ and $\sin(\theta) > 0$
   f. $\tan(\theta) < 0$ and $\cos(\theta) > 0$
   g. $\cos(\theta) < 0$ and $\tan(\theta) > 0$
   h. $\sin(\theta) > 0$ and $\cos(\theta) < 0$

5. Explain why $\sin^2(\theta) + \cos^2(\theta) = 1$.

6. Explain how it is possible to have $\sin(\theta) < 0$, $\cos(\theta) < 0$, and $\tan(\theta) > 0$. For which values of θ between 0 and 2π does this happen?

7. Duncan says that for any real number θ, $\tan(\theta) = \tan(\pi - \theta)$. Is he correct? Explain how you know.

8. Given the following trigonometric functions, identify the quadrant in which the terminal ray of θ lies in the unit circle shown below. Find the other two trigonometric functions of θ of $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.



   a. $\sin(\theta) = \frac{1}{2}$ and $\cos(\theta) > 0$.
   b. $\cos(\theta) = -\frac{1}{2}$ and $\sin(\theta) > 0$.
   c. $\tan(\theta) = 1$ and $\cos(\theta) < 0$.
   d. $\sin(\theta) = -\frac{\sqrt{3}}{2}$ and $\cot(\theta) < 0$.
   e. $\tan(\theta) = -\sqrt{3}$ and $\cos(\theta) < 0$.
   f. $\sec(\theta) = -2$ and $\sin(\theta) < 0$.
   g. $\cot(\theta) = \sqrt{3}$ and $\csc(\theta) > 0$.

9. Toby thinks the following trigonometric equations are true. Use $\theta = \frac{\pi}{6}, \frac{\pi}{4}$, and $\frac{\pi}{3}$ to develop a conjecture whether or not he is correct in each case below.
   a. $\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$.
   b. $\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$.

10. Toby also thinks the following trigonometric equations are true. Is he correct? Justify your answer.
   a. $\sin\left(\boldsymbol{\pi} - \frac{\pi}{3}\right) = \sin(\pi) - \sin\left(\frac{\pi}{3}\right)$
   b. $\cos\left(2\pi - \frac{\pi}{3}\right) = \cos(\pi) - \cos\left(\frac{\pi}{3}\right)$
   c. $\tan\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{6}\right)$
   d. $\sin\left(\pi + \frac{\pi}{6}\right) = \sin(\pi) + \sin\left(\frac{\pi}{6}\right)$
   e. $\cos\left(\pi + \frac{\pi}{4}\right) = \cos(\pi) + \cos\left(\frac{\pi}{4}\right)$