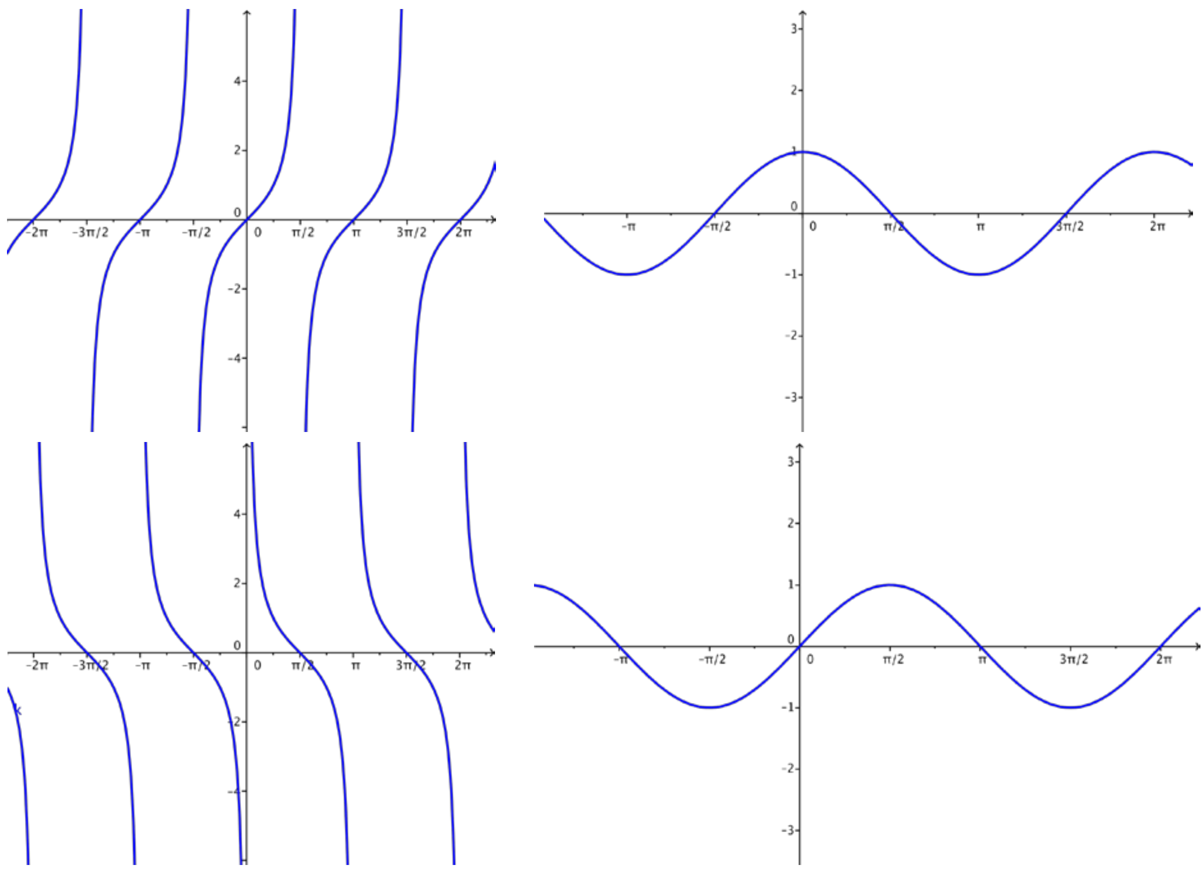
Lesson 2: Properties of Trigonometric Functions

Classwork

Opening Exercise

The graphs below depict four trigonometric functions. Identify which of the graphs are $f(x) = \sin(x)$, $g(x) = \cos(x)$, and $h(x) = \tan(x)$. Explain how you know.



Exercises 1–4

1. Use the unit circle to evaluate these expressions:

   a. $\sin\left(\frac{17\pi}{4}\right)$

   b. $\cos\left(\frac{19\pi}{6}\right)$

   c. $\tan(450\pi)$

2. Use the identity $\sin(\pi+\theta) = -\sin(\theta)$ for all real-numbered values of θ to verify the identity $\sin(2\pi+\theta) = \sin(\theta)$ for all real-numbered values of θ.

3. Use your understanding of the symmetry of the sine and cosine functions to evaluate these functions for the given values of θ.

   a. $\sin(-\frac{\pi}{2})$

   b. $\cos\left(-\frac{5\pi}{3}\right)$

4. Use your understanding of the symmetry of the sine and cosine functions to determine the value of $\tan(-\theta)$ for all real-numbered values of θ. Determine whether the tangent function is even, odd, or neither.

Exploratory Challenge/Exercises 5–6

5. Use your unit circle model to complete the table. Then use the completed table to answer the questions that follow.

θ	$\left(\frac{\pi}{2}+\theta\right)$	$\sin\left(\frac{\pi}{2}+\theta\right)$	$\cos\left(\frac{\pi}{2}+\theta\right)$
0			
$\frac{\pi}{2}$			
π			
$\frac{3\pi}{2}$			
2π			

a. What does the value $\left(\frac{\pi}{2}+\theta\right)$ represent with respect to the rotation of the carousel?

b. What pattern do you recognize in the values of $\sin\left(\frac{\pi}{2}+\theta\right)$ as θ increases from 0 to 2π?

c. What pattern do you recognize in the values of $\cos\left(\frac{\pi}{2}+\theta\right)$ as θ increases from 0 to 2π?

d. Fill in the blanks to formalize these relationships:

$\sin\left(\frac{\pi}{2}+\theta\right) =$

$\cos\left(\frac{\pi}{2}+\theta\right) =$

6. Use your unit circle model to complete the table. Then use the completed table to answer the questions that follow.

$\boldsymbol{\theta}$	$\left(\frac{\pi}{2}-\theta\right)$	$\sin\left(\frac{\pi}{2}-\theta\right)$	$\cos\left(\frac{\pi}{2}-\theta\right)$
0			
$\frac{\pi}{2}$			
π			
$\frac{3\pi}{2}$			
2π			

a. What does the value $\left(\frac{\pi}{2}-\theta\right)$ represent with respect to the rotation of a rider on the carousel?

b. What pattern do you recognize in the values of $\sin\left(\frac{\pi}{2}-\theta\right)$ as θ increases from 0 to 2π?

c. What pattern do you recognize in the values of $\cos\left(\frac{\pi}{2}-\theta\right)$ as θ increases from 0 to 2π?

d. Fill in the blanks to formalize these relationships:

$\sin\left(\frac{\pi}{2} - \theta\right) =$

$\cos\left(\frac{\pi}{2} - \theta\right) =$

Exercise 7

7. Use your understanding of the relationship between the sine and cosine functions to verify these statements.

a. $\cos\left(\frac{4\pi}{3}\right) = \sin\left(\frac{-\pi}{6}\right)$

b. $\cos\left(\frac{5\pi}{4}\right) = \sin\left(\frac{7\pi}{4}\right)$

Lesson Summary

For all real numbers θ for which the expressions are defined,

$\sin(\theta) = \sin(2\pi n + \theta)$ and $\cos(\theta) = \cos(2\pi n + \theta)$ for all integer values of n

$\tan(\theta) = \tan(\pi n + \theta)$ for all integer values of n

$\sin(-\theta) = -\sin(\theta)$, $\cos(-\theta) = \cos(\theta)$, and $\tan(-\theta) = -\tan(\theta)$

$\sin\left(\frac{\pi}{2} + \theta\right) = \cos(\theta)$ and $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin(\theta)$

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$ and $\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$

Problem Set

1. Evaluate the following trigonometric expressions. Show how you used the unit circle to determine the solution.
   - **a.** $\sin\left(\frac{13\pi}{6}\right)$
   - **b.** $\cos\left(-\frac{5\pi}{3}\right)$
   - **c.** $\tan\left(\frac{25\pi}{4}\right)$
   - **d.** $\sin\left(-\frac{3\pi}{4}\right)$
   - **e.** $\cos\left(-\frac{5\pi}{6}\right)$
   - f. $\sin\left(\frac{17\pi}{3}\right)$
   - g. $\cos\left(\frac{25\pi}{4}\right)$
   - h. $\tan\left(\frac{29\pi}{6}\right)$
   - i. $\sin\left(-\frac{31\pi}{6}\right)$
   - j. $\cos\left(-\frac{32\pi}{6}\right)$
   - k. $\tan\left(-\frac{18\pi}{3}\right)$

2. Given each value of β below, find a value of α with $0 \le \alpha \le 2\pi$ so that $\cos(\alpha) = \cos(\beta)$ and $\alpha \ne \beta$.
   a. $\beta = \frac{3\pi}{4}$
   b. $\beta = \frac{5\pi}{6}$
   c. $\beta = \frac{11\pi}{12}$
   d. $\beta = 2\pi$
   e. $\beta = \frac{7\pi}{5}$
   f. $\beta = \frac{17\pi}{30}$
   g. $\beta = \frac{8\pi}{11}$

3. Given each value of β below, find two values of α with $0 \le \alpha \le 2\pi$ so that $\cos(\alpha) = \sin(\beta)$.
   a. $\beta = \frac{\pi}{3}$
   b. $\beta = \frac{7\pi}{6}$
   c. $\beta = \frac{3\pi}{4}$
   d. $\beta = \frac{\pi}{8}$

4. Given each value of β below, find two values of α with $0 \le \alpha \le 2\pi$ so that $\sin(\alpha) = \cos(\beta)$.
   a. $\beta = \frac{\pi}{3}$
   b. $\beta = \frac{5\pi}{6}$
   c. $\beta = \frac{7\pi}{4}$
   d. $\beta = \frac{\pi}{12}$

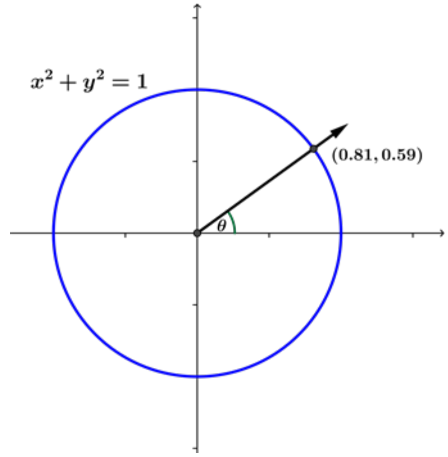
5. Jamal thinks that $\cos\left(\alpha - \frac{\pi}{4}\right) = \sin\left(\alpha + \frac{\pi}{4}\right)$ for any value of α. Is he correct? Explain how you know.

6. Shawna thinks that $\cos\left(\alpha - \frac{\pi}{3}\right) = \sin\left(\alpha + \frac{\pi}{6}\right)$ for any value of α. Is she correct? Explain how you know.

7. Rochelle looked at Jamal and Shawna's results from Problems 5 and 6 and came up with the conjecture below. Is she correct? Explain how you know.

$$\text{Conjecture: } \cos(\alpha - \beta) = \sin\left(\alpha + \left(\frac{\pi}{2} - \beta\right)\right).$$

8. A frog is sitting on the edge of a playground carousel with radius 1 meter. The ray through the frog's position and the center of the carousel makes an angle of measure θ with the horizontal, and his starting coordinates are approximately $(0.81, 0.59)$. Find his new coordinates after the carousel rotates by each of the following amounts.



   a. $\frac{\pi}{2}$
   b. π
   c. 2π
   d. $-\frac{\pi}{2}$
   e. $-\pi$
   f. $\frac{\pi}{2} - \theta$
   g. $\pi - 2\theta$
   h. -2θ