Lesson 7: An Area Formula for Triangles

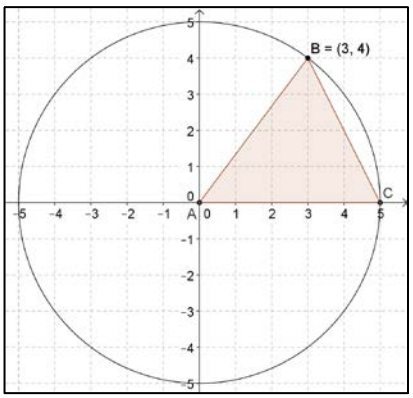
Classwork

Exploratory Challenge\Exercises 1–10: Triangles in Circles

In this Exploratory Challenge, you will find the area of triangles with base along the positive x-axis and a third point on the graph of the circle $x^2 + y^2 = 25$.

1. Find the area of each triangle shown below. Show work to support your answer.

a.



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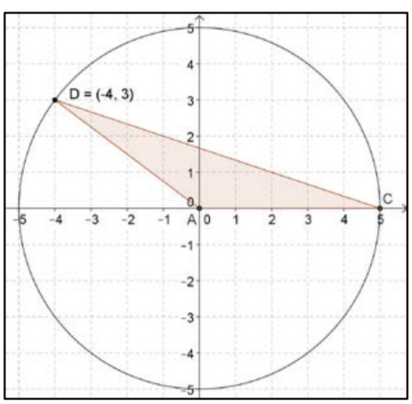
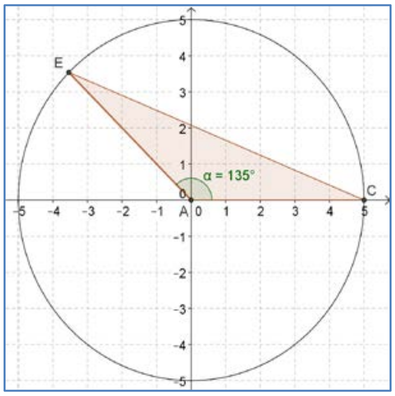


2. Find the area of each of the triangles shown below. Show work to support your answer.

   a.



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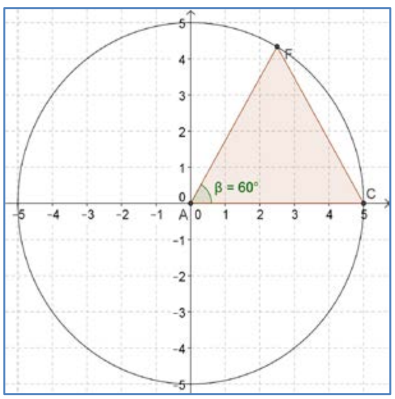


3. Joni said that the area of triangle AFC in Exercise 2, part (b) can be found using the definition of the sine function.

   a. What are the coordinates of point F in terms of the cosine and sine functions? Explain how you know.

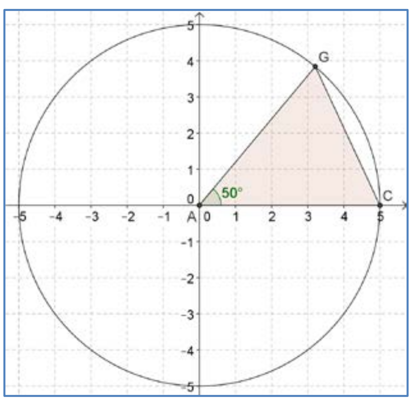
   b. Explain why the y-coordinate of point F is equal to the height of the triangle.

c. Write the area of triangle AFC in terms of the sine function.

d. Does this method work for the area of triangle AEC?

4. Find the area of the following triangles.

a.



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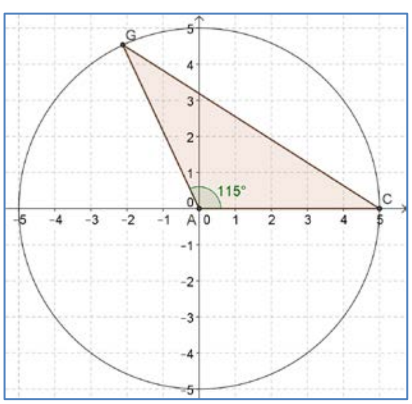
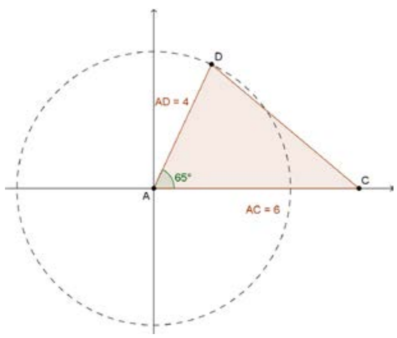


5. Write a formula that will give the area of any triangle with vertices located at $A(0,0)$, $C(5,0)$ and $B(x, y)$ a point on the graph of $x^2 + y^2 = 25$ such that $y > 0$.

6. For what value of θ will this triangle have maximum area? Explain your reasoning.

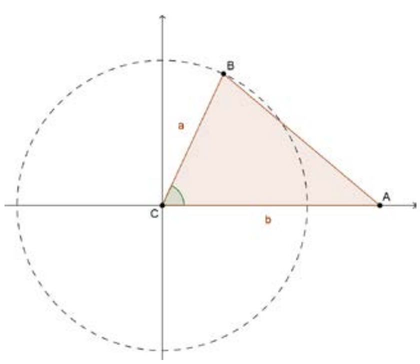
7. Find the area of the following triangle.



8. Prove that the area of any oblique triangle is given by the formula

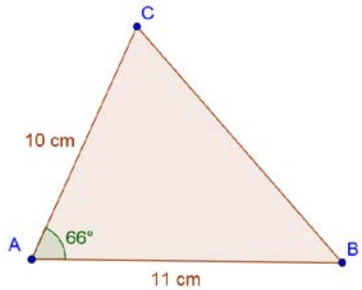
$$\text{Area} = \frac{1}{2}ab\sin(C)$$

where a and b are adjacent sides of ΔABC and C is the measure of the angle between them.



9. Use the area formula from Exercise 8 to calculate the area of the following triangles.

a.



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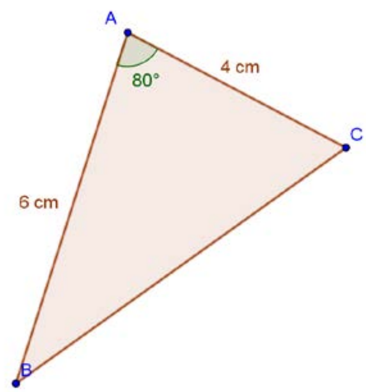
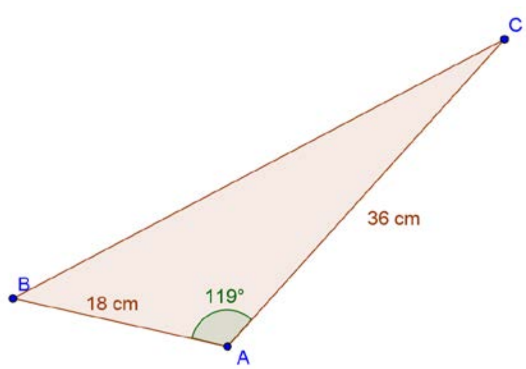
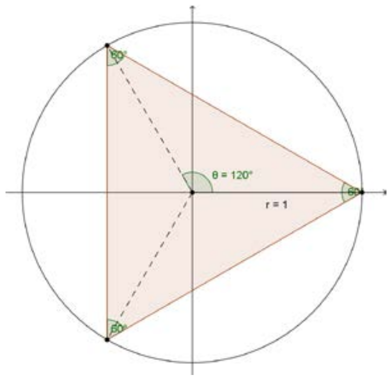


c. A quilter is making an applique design with triangular pieces like the one shown below. How much fabric is used in each piece?

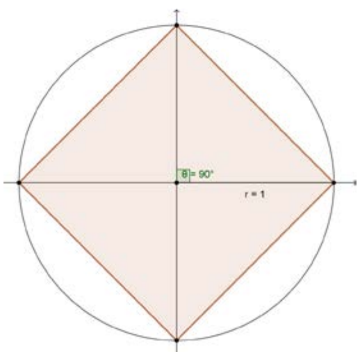


10. Calculate the area of the following regular polygons inscribed in a unit circle by dividing the polygon into congruent triangles where one of the triangles has a base along the positive x-axis.

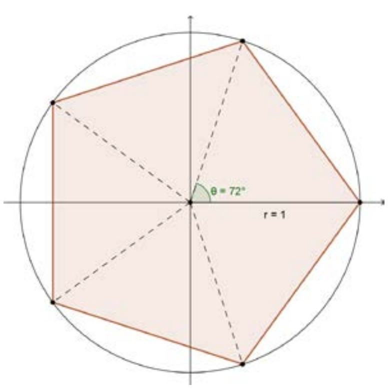
a.



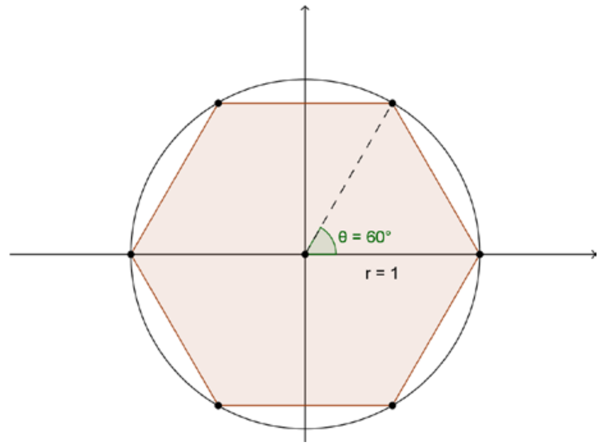
b.



c.



d. Sketch a regular hexagon inscribed in a unit circle with one vertex at $(1,0)$, and find the area of this hexagon.



e. Write a formula that will give the area of a regular polygon with n sides inscribed in a unit circle if one vertex is at $(1,0)$ and θ is the angle formed by the positive x-axis and the segment connecting the origin to the point on the polygon that lies in the first quadrant.

f. Use a calculator to explore the area of this regular polygon for large values of n. What does the area of this polygon appear to be approaching as the value of n increases?

Lesson Summary

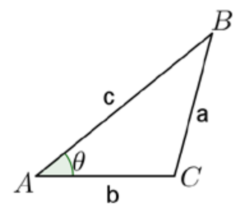
The area of ΔABC is given by the formula:

$$\text{Area} = \frac{1}{2}ab\sin C$$

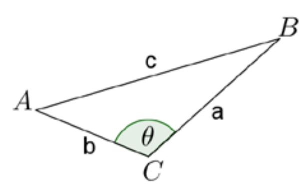
where A and B are the lengths of two sides of the triangle and C is the measure of angle between these sides.

Problem Set

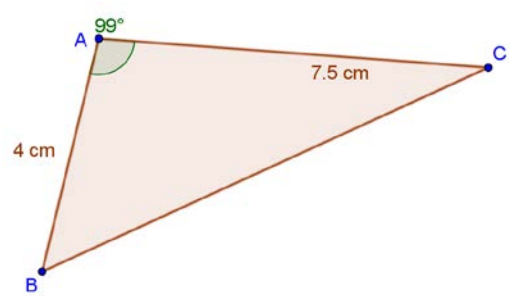
1. Find the area of the triangle ABC shown below, with the following data:
   a. $\theta = \frac{\pi}{6}$, $b = 3$, and $c = 6$.
   b. $\theta = \frac{\pi}{3}$, $b = 4$, and $c = 8$.
   c. $\theta = \frac{\pi}{4}$, $b = 5$, and $c = 10$.



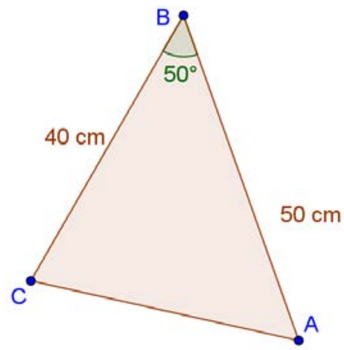
2. Find the area of the triangle ABC shown below, with the following data:
   a. $\theta = \frac{3\pi}{4}$, $a = 6$, and $b = 4$.
   b. $\theta = \frac{5\pi}{6}$, $a = 4$, and $b = 3$.



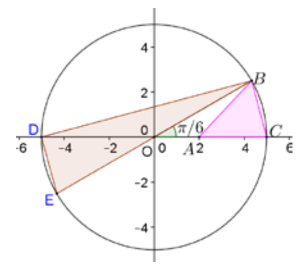
3. Find the area of each triangle shown below. State the area to the nearest tenth of a square centimeter.
   a.



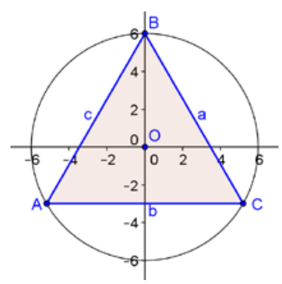
b.



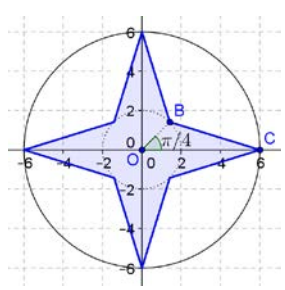
4. The diameter of the circle O in the figure shown below is $\overline{EB} = 10$.
   a. Find the area of the triangle OBA.
   b. Find the area of the triangle ABC.
   c. Find the area of the triangle DBO.
   d. Find the area of the triangle DBE.



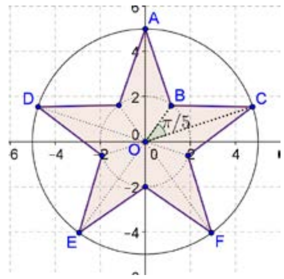
5. Find the area of the equilateral triangle ABC inscribed in a circle with a radius of 6.



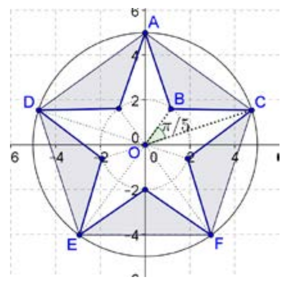
6. Find the shaded area in the diagram below.



7. Find the shaded area in the diagram below. The radius of the outer circle is 5; the length of the line segment $\overline{OB}$ is 2.



8. Find the shaded area in the diagram below. The radius of the outer circle is 5.



9. Find the area of the regular hexagon inscribed in a circle if one vertex is at $(2,0)$.

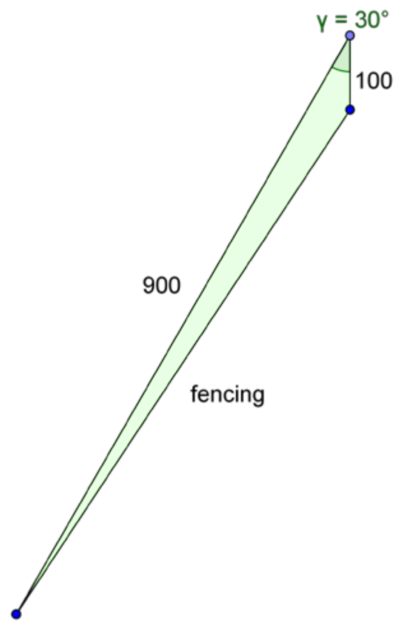
10. Find the area of the regular dodecagon inscribed in a circle if one vertex is at $(3,0)$.

11. A horse rancher wants to add on to existing fencing to create a triangular pasture for colts and fillies. She has 1000 feet of fence to construct the additional two sides of the pasture.

    a. What angle between the two new sides would produce the greatest area?

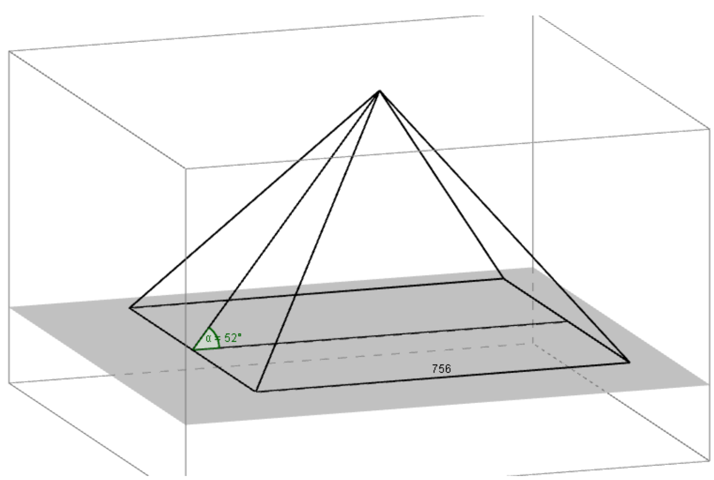
b. What is the area of her pasture if she decides to make two sides of 500 ft. each and uses the angle you found in part (a)?

c. Due to property constraints, she ends up using sides of 100 ft. and 900 ft. with an angle of $30°$ between them. What is the area of the new pasture?



12. An enthusiast of Egyptian history wants to make a life-size version of the Great Pyramid using modern building materials. The base of each side of the Great Pyramid was measured to be 756 ft. long, and the angle of elevation is about $52°$.

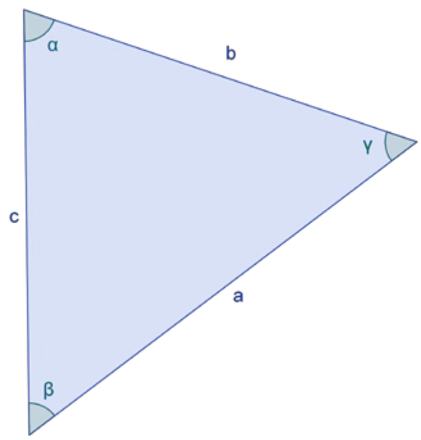
    a. How much material will go into the creation of the sides of the structure (the triangular faces of the pyramid)?



    b. If the price of plywood for the sides is $\$0.75$ per square foot, what is the cost of just the plywood for the sides?

13. Depending on which side you choose to be the "base," there are three possible ways to write the area of an oblique triangle, one being $A = \frac{1}{2}ab\sin(\gamma)$.

    a. Write the other two possibilities using $\sin(\alpha)$ and $\sin(\beta)$.



    b. Are all three equal?

    c. Find $\frac{2A}{abc}$ for all three possibilities.

    d. Is the relationship you found in part (c) true for all triangles?