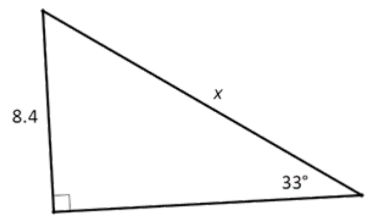
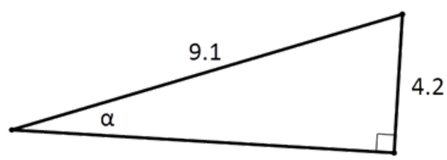
Lesson 8: Law of Sines

Classwork

Exercises

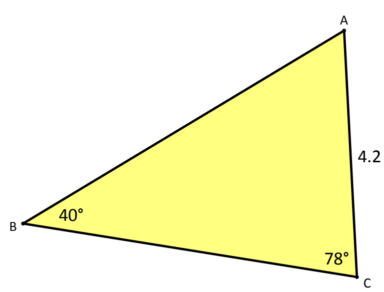
1. Find the value of x in the figure at the left.



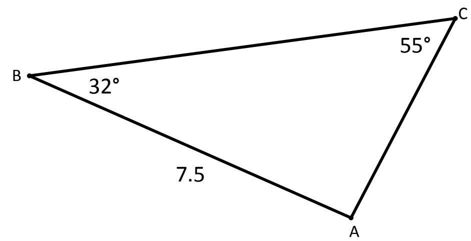


2. Find the value of α in the figure at the right.

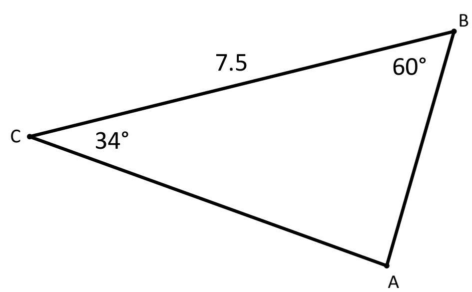
3. Find all of the measurements for the triangle below.



4. Find the length of side AC in the triangle below.



5. A hiker at point C is 7.5 kilometers from a hiker at point B; a third hiker is at point A. Use the angles shown in the diagram above to determine the distance between the hikers at points C and A.



6. Two sides of a triangle have lengths 10.4 and 6.4. The angle opposite 6.4 is $36°$. What could the angle opposite 10.4 be?

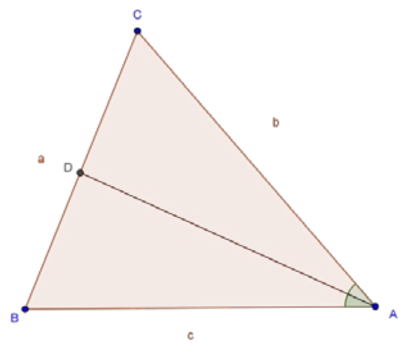
7. Two sides of a triangle have lengths 9.6 and 11.1. The angle opposite 9.6 is $59°$. What could the angle opposite 11.1 be?

Problem Set

1. Let ΔABC be the triangle with the given lengths and angle measurements. Find all possible missing measurements using the law of sines.
   a. $a = 5, m\angle A = 43, m\angle B = 80$.
   b. $a = 3.2, m\angle A = 110, m\angle B = 35$.
   c. $a = 9.1, m\angle A = 70, m\angle B = 95$.
   d. $a = 3.2, m\angle B = 30, m\angle C = 45$.
   e. $a = 12, m\angle B = 29, m\angle C = 31$.
   f. $a = 4.7, m\angle B = 18.8, m\angle C = 72$.
   g. $a = 6, b = 3, m\angle A = 91$.
   h. $a = 7.1, b = 7, m\angle A = 70$.
   i. $a = 8, b = 5, m\angle A = 45$.
   j. $a = 3.5, b = 3.6, m\angle A = 37$.
   k. $a = 9, b = 10.1, m\angle A = 61$.
   l. $a = 6, b = 8, m\angle A = 41.5$.

2. A surveyor is working at a river that flows north to south. From her starting point, she sees a location across the river that is $20°$ north of east from her current position, she labels the position S. She moves 110 feet north and measures the angle to S from her new position, seeing that it is $32°$ south of east.
   a. Draw a picture representing this situation.
   b. Find the distance from her starting position to S.
   c. Explain how you can use the procedure the surveyor used in this problem (called triangulation) to calculate the distance to another object.

3. Consider the triangle pictured below.



Use the law of sines to prove the generalized angle bisector theorem, that is, $\frac{\overline{BD}}{\overline{DC}} = \frac{c\sin(\angle BAD)}{b\sin(\angle CAD)}$. (Although this is called the generalized angle bisector theorem, we do not assume that the angle bisector of BAC intersects side $\overline{BC}$ at D. In the case that AD is an angle bisector, then the formula simplifies to $\frac{\overline{BD}}{\overline{DC}} = \frac{c}{b}$.)

a. Use the triangles ABD and ACD to express $\frac{c}{\overline{BD}}$ and $\frac{b}{\overline{DC}}$ as a ratio of sines.

b. Note that angles BDA and ADC form a linear pair. What does this tell you about the value of the sines of these angles?

c. Solve each equation in part (a) to be equal to the sine of either $\angle BDA$ or $\angle ADC$.

d. What do your answers to parts (b) and (c) tell you?

e. Prove the generalized angle bisector theorem.

4. As an experiment, Carrie wants to independently confirm the distance to Alpha Centauri. She knows that if she measures the angle of Alpha Centauri and waits 6 months and measures again, then she will have formed a massive triangle with two angles and the side between them being 2 AU long.

a. Carrie measures the first angle at $82°\,8'24.5''$ and the second at $97°\,51'34''$. How far away is Alpha Centauri according to Carrie's measurements?

b. Today, astronomers use the same triangulation method on a much larger scale by finding the distance between different spacecraft using radio signals, and then measuring the angles to stars. Voyager 1 is about 122 AU away from Earth. What fraction of the distance from Earth to Alpha Centauri is this? Do you think that measurements found in this manner are very precise?

5. A triangular room has sides of length 3.8, 5.1, and 5.1 m. What is the area of the room?

6. Sara and Paul are on opposite sides of a building that a telephone pole fell on. The pole is leaning away from Paul at an angle of $59°$ and towards Sara. Sara measures the angle of elevation to the top of the telephone pole to be $22°$, and Paul measures the angle of elevation to be $34°$. Knowing that the telephone pole is about 35 ft. tall, answer the following questions.

a. Draw a diagram of the situation.

b. How far apart are Sara and Paul?

c. If we assume the building is still standing, how tall is the building?