# Lesson 11: Revisiting the Graphs of the Trigonometric Functions

## Classwork

### Opening Exercise

Graph each of the following functions on the interval $-2\pi \leq x \leq 4\pi$ by making a table of values. The graph should show all key features (intercepts, asymptotes, relative maxima and minima).

a. $f(x) = \sin(x)$

| | |
|---|---|
| $x$ | |
| $\sin(x)$ | |

b. $f(x) = \cos(x)$

| | |
|---|---|
| $x$ | |
| $\cos(x)$ | |

## Exercises 1–7

1. Consider the trigonometric function $f(x) = \tan(x)$.

   a. Rewrite $\tan(x)$ as a quotient of trigonometric functions. Then, state the domain of the tangent function.

   b. Why is this the domain of the function?

   c. Complete the table.

| $x$ | $-2\pi$ | $-\frac{3\pi}{2}$ | $-\pi$ | $-\frac{\pi}{2}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ | $\frac{5\pi}{2}$ | $3\pi$ | $\frac{7\pi}{2}$ | $4\pi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\tan(x)$ | | | | | | | | | | | | | |

   d. What will happen on the graph of $f(x) = \tan(x)$ at the values of $x$ for which the tangent function is undefined?

   e. Expand the table to include angles that have a reference angle of $\frac{\pi}{4}$.

| $x$ | $-\frac{7\pi}{2}$ | $-\frac{5\pi}{2}$ | $-\frac{3\pi}{2}$ | $-\frac{\pi}{2}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{2}$ | $\frac{5\pi}{2}$ | $\frac{7\pi}{2}$ | $\frac{9\pi}{2}$ | $\frac{11\pi}{2}$ | $\frac{13\pi}{2}$ | $\frac{15\pi}{2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\tan(x)$ | | | | | | | | | | | | |

f. Sketch the graph of $f(x) = \tan(x)$ on the interval $-2\pi \leq x \leq 4\pi$. Verify by using a graphing utility.

2. Use the graphs of the sine, cosine, and tangent functions to answer each of the following.

a. How do the graphs of the sine and cosine functions support the following identities for all real numbers $x$?

$$\sin(-x) = -\sin(x)$$
$$\cos(-x) = \cos(x)$$

b. Use the symmetry of the graph of the tangent function to write an identity. Explain your answer.

c. How do the graphs of the sine and cosine functions support the following identities for all real numbers $x$?

$$\sin(x + 2\pi) = \sin(x)$$
$$\cos(x + 2\pi) = \cos(x)$$

d. Use the periodicity of the tangent function to write an identity. Explain your answer.

3. Consider the function $f(x) = \cos\left(x - \frac{\pi}{2}\right)$.

a. Graph $y = f(x)$ by using transformations of functions.

b. Based on your graph, write an identity.

4. Verify the identity $\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$ for all real numbers $x$ by using a graph.

5. Use a graphing utility to explore the graphs of the family of functions in the form $f(x) = A\sin\big(\omega(x-h)\big) + k$. Write a summary of the effect that changing each parameter has on the graph of the sine function.
   a. $A$

   b. $\omega$

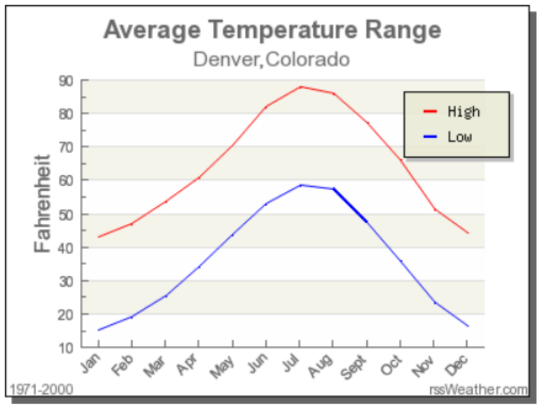
   c. $h$

   d. $k$

6. Graph at least one full period of the function $f(x) = 3\sin\left(\frac{1}{3}(x-\pi)\right) + 2$. Label the amplitude, period, and midline on the graph.

7. The graph and table below show the average monthly high and low temperature for Denver, Colorado. (source: http://www.rssweather.com/climate/Colorado/Denver/)

**Average Temperatures for Denver**




| Month | Low | High |
|---|---|---|
| Jan | 15.2°F | 43.2°F |
| Feb | 19.1°F | 47.2°F |
| Mar | 25.4°F | 53.7°F |
| Apr | 34.2°F | 60.9°F |
| May | 43.8°F | 70.5°F |
| Jun | 53.0°F | 82.1°F |
| Jul | 58.7°F | 88.0°F |
| Aug | 57.4°F | 86.0°F |
| Sept | 47.3°F | 77.4°F |
| Oct | 35.9°F | 66.0°F |
| Nov | 23.5°F | 51.5°F |
| Dec | 16.4°F | 44.1°F |

a. Why would a sinusoidal function be appropriate to model this data?

b. Write a function to model the average monthly high temperature as a function of the month.

c. What does the midline represent within the context of the problem?

d. What does the amplitude represent within the context of the problem?

e. Name a city whose temperature graphs would have a smaller amplitude. Explain your reasoning.

f. Name a city whose temperature graphs would have a larger vertical shift. Explain your reasoning.

## Problem Set

1. Sketch the graph of $y = \sin(x)$ on the same set of axes as the function $f(x) = \sin(4x)$. Explain the similarities and differences between the two graphs.

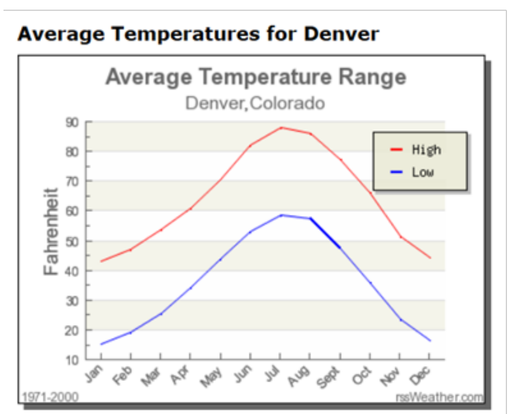
2. Sketch the graph of $y = \sin\left(\frac{x}{2}\right)$ on the same set of axes as the function $g(x) = 3\sin\left(\frac{x}{2}\right)$. Explain the similarities and differences between the two graphs.

3. Indicate the amplitude, frequency, period, phase shift, horizontal and vertical translations, and equation of the midline. Graph the function on the same axes as the graph of the cosine function $f(x) = \cos(x)$. Graph at least one full period of each function.

$$g(x) = \cos\left(x - \frac{3\pi}{4}\right).$$

4. Sketch the graph of the pairs of functions on the same set of axes: $f(x) = \sin(4x)$, $g(x) = \sin(4x) + 2$.

5. The graph and table below show the average monthly high and low temperature for Denver, Colorado. (source: http://www.rssweather.com/climate/Colorado/Denver/)

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| Oct | 35.9°F | 66.0°F |
| Nov | 23.5°F | 51.5°F |
| Dec | 16.4°F | 44.1°F |

Write a function to model the average monthly low temperature as a function of the month.

**Extension:**

6. Consider the cosecant function.
   a. Use technology to help you sketch $y = \csc(x)$ for $0 \le x \le 4\pi$, $-4 \le y \le 4$.
   b. What do you notice about the graph of the function? Compare this to your knowledge of the graph of $y = \sin(x)$.

7. Consider the secant function.
   a. Use technology to help you sketch $y = \sec(x)$ for $0 \leq x \leq 4\pi$, $-4 \leq y \leq 4$.
   b. What do you notice about the graph of the function? Compare this to your knowledge of the graph of $y = \cos(x)$.

8. Consider the cotangent function.
   a. Use technology to help you sketch $y = \cot(x)$ for $0 \leq x \leq 2\pi$, $-4 \leq y \leq 4$.
   b. What do you notice about the graph of the function? Compare this to your knowledge of the graph of $y = \tan(x)$.