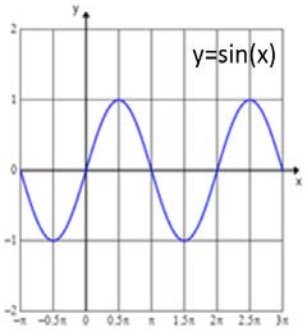
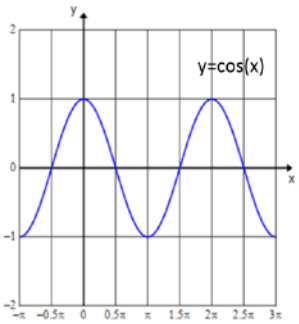
Lesson 12: Inverse Trigonometric Functions

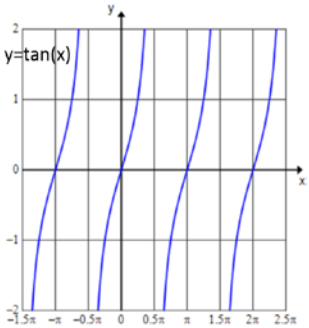
Classwork

Opening Exercise

Use the graphs of the sine, cosine, and tangent functions to answer each of the following questions.







a. State the domain of each function.

b. Would the inverse of the sine, cosine, or tangent functions also be functions? Explain.

c. For each function, select a suitable domain that will make the function invertible.

Example 1

Consider the function $f(x) = \sin(x), -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

a. State the domain and range of this function.

b. Find the equation of the inverse function.

c. State the domain and range of the inverse.

Exercises 1–3

1. Write an equation for the inverse cosine function, and state its domain and range.

2. Write an equation for the inverse tangent function, and state its domain and range.

3. Evaluate each of the following expressions without using a calculator. Use radian measures.

a. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

b. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

c. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

d. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

e. $\sin^{-1}(1)$

f. $\sin^{-1}(-1)$

g. $\cos^{-1}(1)$

h. $\cos^{-1}(-1)$

i. $\tan^{-1}(1)$

j. $\tan^{-1}(-1)$

Example 2

Solve each trigonometric equation such that $0 \leq x \leq 2\pi$. Round to three decimal places when necessary.

a. $2\cos(x) - 1 = 0$

b. $3\sin(x) + 2 = 0$

Exercises 4–8

4. Solve each trigonometric equation such that $0 \leq x \leq 2\pi$. Give answers in exact form.

   a. $\sqrt{2}\cos(x) + 1 = 0$

   b. $\tan(x) - \sqrt{3} = 0$

   c. $\sin^2(x) - 1 = 0$

5. Solve each trigonometric equation such that $0 \leq x \leq 2\pi$. Round answers to three decimal places.

   a. $5\cos(x) - 3 = 0$

   b. $3\cos(x) + 5 = 0$

c. $3\sin(x) - 1 = 0$

d. $\tan(x) = -0.115$

6. A particle is moving along a straight line for $0 \leq t \leq 18$. The velocity of the particle at time t is given by the function $v(t) = \cos\left(\frac{\pi}{5}t\right)$. Find the time(s) on the interval $0 \leq t \leq 18$ where the particle is at rest ($v(t) = 0$).

7. In an amusement park, there is a small Ferris wheel, called a kiddie wheel, for toddlers. The formula $H(t) = 10\sin\left(2\pi\left(t - \frac{1}{4}\right)\right) + 15$ models the height H (in feet) of the bottom-most car t minutes after the wheel begins to rotate. Once the ride starts, it lasts 4 minutes.

   a. What is the initial height of the car?

   b. How long does it take for the wheel to make one full rotation?

   c. What is the maximum height of the car?

d. Find the time(s) on the interval $0 \leq t \leq 4$ when the car is at its maximum height.

8. Many animal populations fluctuate periodically. Suppose that a wolf population over an 8-year period is given by the function $W(t) = 800\sin\left(\frac{\pi}{4}t\right) + 2200$, where t represents the number of years since the initial population counts were made.

a. Find the time(s) on the interval $0 \leq t \leq 8$ such that the wolf population equals 2500.

b. On what time interval during the 8-year period is the population below 2000?

c. Why would an animal population be an example of a periodic phenomenon?

Problem Set

1. Solve the following equations. Approximate values of the inverse trigonometric functions to the thousandths place, where x refers to an angle measured in radians.
   a. $5 = 6\cos(x)$
   b. $-\frac{1}{2} = 2\cos\left(x - \frac{\pi}{4}\right) + 1$
   c. $1 = \cos\big(3(x-1)\big)$
   d. $1.2 = -0.5\cos(\pi x) + 0.9$
   e. $7 = -9\cos(x) - 4$
   f. $2 = 3\sin(x)$
   g. $-1 = \sin\left(\frac{\pi(x-1)}{4}\right) - 1$
   h. $\pi = 3\sin(5x+2) + 2$
   i. $\frac{1}{9} = \frac{\sin(x)}{4}$
   j. $\cos(x) = \sin(x)$
   k. $\sin^{-1}(\cos(x)) = \frac{\pi}{3}$
   l. $\tan(x) = 3$
   m. $-1 = 2\tan(5x+2) - 3$
   n. $5 = -1.5\tan(-x) - 3$

2. Fill out the following tables.

x	$\sin^{-1}(x)$	$\cos^{-1}(x)$
-1		
$-\frac{\sqrt{3}}{2}$		
$-\frac{\sqrt{2}}{2}$		
$-\frac{1}{2}$		

x	$\sin^{-1}(x)$	$\cos^{-1}(x)$
0		
$\frac{1}{2}$		
$\frac{\sqrt{2}}{2}$		
$\frac{\sqrt{3}}{2}$		
1		

3. Let the velocity v in miles per second of a particle in a particle accelerator after t seconds be modeled by the function $v = \tan\left(\frac{\pi t}{6000} - \frac{\pi}{2}\right)$ on an unknown domain.
   a. What is the t-value of the first vertical asymptote to the right of the y-axis?
   b. If the particle accelerates to 99% of the speed of light before stopping, then what is the domain? Note: $c \approx 186000$. Round your solution to the ten-thousandths place.
   c. How close does the domain get to the vertical asymptote of the function?
   d. How long does it take for the particle to reach the velocity of Earth around the sun (about 18.5 miles per second)?
   e. What does it imply that v is negative up until $t = 3000$?