Lesson 13: Modeling with Inverse Trigonometric Functions

Classwork

Example

The Statue of Liberty is 151 feet tall and sits on a pedestal that is 154 feet above the ground. An observer who is 6 feet tall wants to stand at the ideal viewing distance in front of the statue.

a. Sketch the statue and observer. Label all appropriate measurements on the sketch, and define them in context.

b. How far back from the statue should the observer stand so that his or her viewing angle (from the feet of the statue to the tip of the torch) is largest? What is the value of the largest viewing angle?

c. What would be your best viewing distance from the statue?

d. If there are 66 meters of dry land in front of the statue, is the viewer still on dry land at the best viewing distance?

Exercise

Hanging on a museum wall is a picture with base a inches above a viewer's eye level and top b inches above the viewer's eye level.

a. Model the situation with a diagram.

b. Determine an expression that could be used to find the ideal viewing distance x that maximizes the viewing angle y.

c. Find the ideal viewing distance, given the a and b values assigned to you. Calculate the maximum viewing angle in degrees.

d. Complete the table using class data, which indicates the ideal values for x given different assigned values of a and b. Note any patterns you see in the data.

a (inches)	b (inches)	x at max (inches)	y max (degrees)

Problem Set

1. Consider the situation of sitting down with eye level at 46 in.. Find the missing distances and heights for the following:
   a. The bottom of the picture is at 50 in. and the top is at 74 in. What is the optimal viewing distance?
   b. The bottom of the picture is at 52 in. and the top is at 60 in. What is the optimal viewing distance?
   c. The bottom of the picture is at 48 in. and the top is at 64 in. What is the optimal viewing distance?
   d. What is the height of the picture if the optimal viewing distance is 1 ft. and the bottom of the picture is hung at 47 in.?

2. Consider the situation where you are looking at a painting a inches above your line of sight and b inches below your line of sight.
   a. Find the optimal viewing distance if it exists.
   b. If the average standing eye height of Americans is 61.4 in., at what height should paintings and other works of art be hung?

3. The amount of daylight per day is periodic with respect to the day of the year. The function $y = -3.016\cos\left(\frac{2\pi x}{365}\right) + 12.25$ gives the number of hours of daylight in New York, y, as a function of the number of days since the winter solstice (December 22), which is represented by x.
   a. On what days will the following hours of sunlight occur?
      i. 15 hours, 15 minutes.
      ii. 12 hours.
      iii. 9 hours, 15 minutes.
      iv. 10 hours.
      v. 9 hours.
   b. Give a function that will give the day of the year from the solstice as a function of the hours of daylight.
   c. What is the domain of the function you gave in part (b)?
   d. What does the domain tell you in the context of the problem?
   e. What is the range of the function? Does this make sense in the context of the problem? Explain.

4. Ocean tides are an example of periodic behavior. At a particular harbor, data was collected over the course of 24 hours to create the following model: $y = 1.236\sin\left(\frac{\pi}{3}x\right) + 1.798$, which gives the water level, y, in feet above the MLLW (mean lower low water) as a function of the time, x, in hours.
   a. How many periods are there each day?
   b. Write a function that gives the time in hours as a function of the water level. How many other times per day will have the same water levels as those given by the function?