Name $\qquad$ Date $\qquad$

## Lesson 1: The General Multiplication Rule

## Exit Ticket

Serena is in a math class of 20 students. Each day for a week (Monday to Friday), a student in Serena's class is randomly selected by the teacher to explain a homework problem. Once a student's name is selected, that student is not eligible to be selected again that week.
1.
a. What is the probability that Serena is selected on Monday?
b. What is the probability that Serena is selected on Tuesday given that she was not selected on Monday?
c. What is the probability that she will be selected on Friday given that she was not selected on any of the other days?
2. Suppose $A$ represents Serena being selected, and $B$ represents Dominic (another student in class) being selected. The event $A 1$ means Serena was selected on Monday, and the event $B 2$ means Dominic was selected on Tuesday. The event $B 1$ means Dominic was selected on Monday, and the event $A 2$ means Serena was selected on Tuesday.
a. Explain in words what $P(A 1$ and $B 2)$ represents, and then calculate this probability.
b. Explain in words what $P(B 1$ and $A 2)$ represents, and then calculate this probability.
$\qquad$

## Lesson 2: Counting Rules-The Fundamental Counting Principle and Permutations

## Exit Ticket

1. The combination for the lock shown below consists of three numbers.
a. If the numbers can be repeated, how many different combinations are there? Explain your answer.
b. If the numbers cannot be repeated, how many different combinations are there? Explain your answer.

2. Jacqui is putting together sets of greeting cards for a school fundraiser. There are four different card options, two different colored envelopes, and four different sticker designs. A greeting card set consists of one type of card, one color for the envelopes, and one sticker design. How many different ways can Jacqui arrange the greeting card sets? Explain how you determined your answer.

Name $\qquad$ Date $\qquad$

## Lesson 3: Counting Rules-Combinations

## Exit Ticket

1. Timika is a counselor at a summer camp for young children. She wants to take 20 campers on a hike and wants to choose a pair of students to lead the way. In how many ways can Timika choose this pair of children?
2. Sean has 56 songs on his MP3 player. He wants to randomly select 6 of the songs to use in a school project. How many different groups of 6 songs could Sean select? Did you calculate the number of permutations or the number of combinations to get your answer? Why did you make this choice?
3. A fast food restaurant has the following options for toppings on their hamburgers: mustard, ketchup, mayo, onions, pickles, lettuce, and tomato. In how many ways could a customer choose four different toppings from these options?
4. Seven colored balls (red, blue, yellow, black, brown, white, and orange) are in a bag. A sample of three balls is selected without replacement. How many different samples are possible?

Name
Date $\qquad$

## Lesson 4: Using Permutations and Combinations to Compute

## Probabilities

## Exit Ticket

1. An ice-cream shop has 25 different flavors of ice cream. For each of the following, indicate whether it is a problem that involves permutations, combinations, or neither.
a. What is the number of different 3-scoop ice-cream cones that are possible if all three scoops are different flavors, and a cone with vanilla, strawberry, and chocolate is different from a cone with vanilla, chocolate, and strawberry?
b. What is the number of different 3-scoop ice-cream cones that are possible if all three scoops are different flavors, and a cone with vanilla, strawberry, and chocolate is considered the same as a cone with vanilla, chocolate, and strawberry?
c. What is the number of different ice-cream cones if all three scoops could be the same, and the order of the flavors is important?
2. A train consists of an engine at the front, a caboose at the rear, and 27 boxcars that are numbered from 1 to 27 .
a. How many different orders are there for cars that make up the train?
b. If the cars are attached to the train in a random order, what is the probability that the boxcars are in numerical order from 1 to 27 ?
3. The dance club at school has 22 members. The dance coach wants to send four members to a special training on new dance routines.
a. The dance coach will select four dancers to go to the special training. Is the number of ways to select four dancers a permutation, a combination, or neither? Explain your answer.
b. If the dance coach chooses at random, how would you determine the probability of selecting dancers Laura, Matthew, Lakiesha, and Santos?

Name $\qquad$ Date $\qquad$

## Lesson 5: Discrete Random Variables

## Exit Ticket

1. Create a table that illustrates the probability distribution of a discrete random variable with four outcomes.

| Random variable | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| Probability |  |  |  |  |

2. Which of the following variables are discrete and which are continuous? Explain your answer.

Number of items purchased by a customer at a grocery store
Time required to solve a puzzle
Length of a piece of lumber
Number out of 10 customers who pay with a credit card

## Template for apartment cards used in Example 1

| Apartment number: 1 <br> Number of bedrooms: 1 <br> Size (sq. ft.): 1,102 <br> Color of walls: white <br> Floor number: 1 <br> Distance to elevator: 5 ft . <br> Floor type: carpet | Apartment number: 2 <br> Number of bedrooms: 2 <br> Size (sq. ft.): 975.5 <br> Color of walls: white <br> Floor number: 6 <br> Distance to elevator: 30 ft . <br> Floor type: carpet |
| :---: | :---: |
| Apartment number: 3 <br> Number or bedrooms: 3 <br> Size (sq. ft.): 892.25 <br> Color of walls: green <br> Floor number: 2 <br> Distance to elevator: 20 ft . <br> Floor type: tile | Apartment number: 4 <br> Number or bedrooms: 3 <br> Size (sq. ft.): 639 <br> Color of walls: white <br> Floor number: 2 <br> Distance to elevator: 20.5 ft . <br> Floor type: tile |
| Apartment number: 5 <br> Number or bedrooms: 2 <br> Size (sq. ft.): 2,015 <br> Color of walls: white <br> Floor number: 3 <br> Distance to elevator: 45.25 ft . <br> Floor type: carpet | Apartment number: 6 <br> Number or bedrooms: 2 <br> Size (sq. ft.): 415 <br> Color of walls: white <br> Floor number: 1 <br> Distance to elevator: 40 ft . <br> Floor type: carpet |
| Apartment number: 7 <br> Number or bedrooms: 1 <br> Size (sq. ft.): 1,304 <br> Color of walls: white <br> Floor number: 4 <br> Distance to elevator: 15.75 ft . <br> Floor type: carpet | Apartment number: 8 <br> Number or bedrooms: 2 <br> Size (sq. ft.): 1,500 <br> Color of walls: green <br> Floor number: 3 <br> Distance to elevator: 60.75 ft . <br> Floor type: carpet |
| Apartment number: 9 <br> Number or bedrooms: 2 <br> Size (sq. ft.): 2,349.75 <br> Color of walls: white <br> Floor number: 5 <br> Distance to elevator: 100 ft . <br> Floor type: carpet | Apartment number: 10 <br> Number or bedrooms: 3 <br> Size (sq. ft.): 750 <br> Color of walls: green <br> Floor number: 1 <br> Distance to elevator: 10.5 ft . <br> Floor type: tile |

Name $\qquad$ Date $\qquad$

## Lesson 6: Probability Distribution of a Discrete Random

## Variable

## Exit Ticket

The following statements refer to a discrete probability distribution for the number of songs a randomly selected high school student downloads in a week, according to an online music library.

Probability distribution of number of songs downloaded by high school students in a week:

| Number of <br> songs | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.06 | 0.14 | 0.22 | 0.25 | 0.15 | 0.09 | 0.05 | 0.024 | 0.011 | 0.005 |

Which of the following statements seem reasonable to you based on a random sample of 200 students? Explain your reasoning, particularly for those that are unreasonable.
a. 25 students downloaded 3 songs a week.
b. More students downloaded 4 or more songs than downloaded 3 songs.
c. 30 students in the sample downloaded 9 or more songs per week.

Name $\qquad$ Date $\qquad$

## Lesson 7: Expected Value of a Discrete Random Variable

## Exit Ticket

At a carnival, one game costs $\$ 1$ to play. The contestant gets one shot in an attempt to bust a balloon. Each balloon contains a slip of paper with one of the following messages.

- Sorry, you do not win, but you get your dollar back. (The contestant has not lost the $\$ 1$ cost.)
- Congratulations, you win $\$ 2$. (The contestant has won $\$ 1$.)
- Congratulations, you win \$5. (The contestant has won \$4.)
- Congratulations, you win \$10. (The contestant has won \$9.)

If the contestant does not bust a balloon, then the $\$ 1$ cost is forfeited. The table below displays the probability distribution of the discrete random variable, or net winnings for this game.

| Net winnings | -1 | 0 | 1 | 4 | 9 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.25 | $?$ | 0.3 | 0.08 | 0.02 |

1. What is the sum of the probabilities in a discrete probability distribution? Why?
2. What is the probability that a contestant will bust a balloon and receive the message, "Sorry, you do not win, but you get your dollar back"?
3. What is the net amount that a contestant should expect to win per game if the game were to be played many times?

Name $\qquad$ Date $\qquad$

## Lesson 8: Interpreting Expected Value

## Exit Ticket

At a large university, students are allowed to register for no more than 7 classes. The number of classes for which a student is registered is a discrete random variable. The expected value of this random variable for students at this university is 4.15 .

Write an interpretation of this expected value.
$\qquad$

## Lesson 9: Determining Discrete Probability Distributions

## Exit Ticket

Suppose that an estimated $10 \%$ of the inhabitants of a large island have a certain gene. If pairs of islanders are selected at random and tested for the gene, what is the probability that one or both islanders are carriers? Explain your answer using a probability distribution.

Name $\qquad$ Date $\qquad$

## Lesson 10: Determining Discrete Probability Distributions

## Exit Ticket

$23 \%$ of the cars a certain automaker manufactures are silver. Below is the probability distribution for the number of silver cars sold by a car dealer in the next five car sales.

| Number of <br> Silver Cars | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.27068 | 0.40426 | 0.24151 | 0.07214 | 0.01077 | 0.00064 |

1. What is the probability of selling at most three silver cars? Interpret this probability in context.
2. What is the probability of selling between one and four silver cars? Interpret this probability in context.
3. How many silver cars is the dealer expected to sell, on average, out of five cars? Interpret this expected value in context.

Name $\qquad$ Date $\qquad$

## Lesson 11: Estimating Probability Distributions Empirically

## Exit Ticket

The table shows the number of hours, to the nearest half-hour per day, that teens spend texting, according to a random sample of 870 teenagers aged 13-18 in a large urban city.

Table: Number of hours teenagers spend texting per day

| Hours | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 170 | 82 | 220 | 153 | 92 | 58 | 40 | 15 | 12 | 18 | 10 |

1. What random variable is of interest here? What are the possible values for the random variable?
2. Create an estimated probability distribution for the time teens spend texting.
3. What is the estimated probability that teens spend less than an hour per day texting?
4. Would you be surprised if the average texting time for a smaller random sample of teens in the same city was three hours? Why or why not?

Name $\qquad$ Date $\qquad$

## Lesson 12: Estimating Probability Distributions Empirically

## Exit Ticket

A bus company has 9 seats on a shuttle between two cities, but about $10 \%$ of the time people do not show up for the bus even though they reserve a seat. The company compensates by reserving 11 seats instead of 9 . Assume that whether or not a person with a reservation shows up is independent of what happens with the other reservation holders.
a. Consider the random variable number of people who are denied a seat because more than 9 people showed up for the shuttle. What are the possible values of this random variable?
b. The table displays the number of people who reserved tickets but did show up based on simulating 50 trips between the two cities. Use the information to estimate a probability distribution of the number of people denied a seat on the shuttle.

Table: Number of people who showed up for their reservation

| 10 | 10 | 11 | 9 | 10 | 11 | 9 | 9 | 11 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 10 | 10 | 8 | 9 | 10 | 11 | 9 | 10 | 10 |
| 8 | 11 | 11 | 10 | 11 | 10 | 10 | 11 | 9 | 11 |
| 11 | 10 | 8 | 11 | 9 | 11 | 9 | 10 | 11 | 9 |
| 10 | 9 | 9 | 10 | 10 | 9 | 9 | 10 | 10 | 9 |

c. In the long run, how many people should the company expect to be denied a seat per shuttle trip? Explain how you determined the answer.

Table of Random Numbers

00000110010001100011
10111001111010001100
01101001011011010101
11011011010100110010
01110001000100110011
00010000001100111011
11101001010010000110
10110001110001000100
11101100101101100110
11100010010011100011

Name $\qquad$ Date $\qquad$

1. In the game of tennis, one player serves to a second player to start a point. If the server misses landing the first serve (the ball is hit out of bounds and cannot be played), the server is allowed a second serve. Suppose a particular tennis player, Chris, has a 0.62 probability of a first serve landing in bounds and a 0.80 probability of a second serve landing in bounds. Once a serve has landed in bounds, the players take turns hitting the ball back and forth until one player hits the ball out of bounds or into the net. The player who hits the ball out or into the net loses the point, and the other player wins the point. For Chris, when the first serve lands in bounds, he has a $75 \%$ chance of winning the point; however, he has only a $22 \%$ chance of winning the point on his second serve.
a. Calculate the probability that Chris lands his first serve in bounds and then goes on to win the point for a randomly selected point.
b. Calculate the probability that Chris misses the first serve, lands the second serve, and then wins the point.
c. Calculate the probability that Chris wins a randomly selected point.
2. In Australia, there are approximately 100 species of venomous snakes, but only about 12 have a deadly bite. Suppose a zoo randomly selects one snake from each of five different species to show visitors. What is the probability that exactly one of the five snakes shown is deadly?
3. In California (CA), standard license plates are currently of the form: 1 number-3 letters-3 numbers. Assume the numbers are $0-9$ and the letters are $\mathrm{A}-\mathrm{Z}$.
a. In theory, how many different possible standard CA license plates are there, assuming we can repeat letters and numbers?
b. How many different possible standard CA license plates are there if we are not allowed to repeat any letters or numbers?
c. For part (b), did you use permutations or combinations to carry out the calculation? Explain how you know.
4. When there is a problem with a computer program, many people call a technical support center for help. Several people may call at once, so the center needs to be able to have several telephone lines available at the same time. Suppose we want to consider a random variable that is the number of telephone lines in use by the technical support center of a software manufacturer at a particular time of day. Suppose that the probability distribution of this random variable is given by the following table:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p(x)$ | 0.35 | 0.20 | 0.15 | 0.15 | 0.10 | 0.05 |

a. Produce a graph of the probability distribution for this random variable, including all relevant labels.
b. Calculate the expected value of the random variable.
c. Explain how to interpret this expected value in context.
5. The following table lists the number of U.S. households (in thousands) with 0 vehicles, 1 vehicle, 2 vehicles, or 3 or more vehicles for all households responding to the 2009 National Household Travel Survey.

| No vehicle | One vehicle | Two vehicles | Three or more vehicles |
| :---: | :---: | :---: | :---: |
| 9,828 | 36,509 | 41,077 | 25,668 |

a. Use these data to create a table of relative frequencies that could be used as estimates of the probability distribution of number of vehicles for a randomly selected U.S. household (that responded to the survey).

| No vehicle | One vehicle | Two vehicles | Three or more vehicles |
| :---: | :---: | :---: | :---: |
|  |  |  |  |

b. Suppose you want to examine the distribution of the number of vehicles in all U.S. households. Define a random variable that corresponds to the probability distribution in part (a).
c. Assume for the moment that the last column corresponds to exactly three vehicles. Calculate the expected number of vehicles per household.
d. Now reconsider the last category. Suppose we were to find the information for the actual number of vehicles for these households. Would the expected number of vehicles per household with this new information be larger or smaller than the expected value you found in (c)? Explain your reasoning.
e. Suppose a town has about 4,500 households. What is a good estimate for the number of cars in town? Explain how you determined your answer.

Name
Date $\qquad$

## Lesson 13: Games of Chance and Expected Value

## Exit Ticket

As posted on the Maryland Lottery's website for its Pick 3 game, the chance of winning with a Front Pair bet is 0.01 .
A Front Pair bet is successful if the front pair of numbers you select match the Pick 3 number's first 2 digits. For example, a bet of $12 X$ would be a winner if the Pick 3 number is $120,121,122$, etc. In other words, 10 of the 1,000 possible Pick 3 numbers ( $1 \%$ ) would be winners, and thus, the probability of winning is 0.01 or $1 \%$.

A successful bet of $\$ 0.50$ pays out $\$ 25.00$ for a net gain to the player of $\$ 24.50$.
a. Define the random variable $X$ and compute $E(X)$.
b. On average, how much does the Maryland Lottery make on each such bet?
c. Assume that for a given time period, 100,000 bets like the one described above were placed. How much money should the Maryland Lottery Agency expect to earn on average from 100,000 bets?

Note: According to the Maryland Lottery Gaming and Control Agency's Annual Report for Fiscal Year 2012, the Pick 3 game accounted for $\$ 254.60$ million in net sales. (http://mlgca.com/annual-report/ accessed November 17, 2013)

Name $\qquad$ Date $\qquad$

## Lesson 14: Games of Chance and Expected Value

## Exit Ticket

In the previous lesson, you examined the Maryland Lottery's Pick 3 game where the chance of winning with a Front Pair bet is 0.01 . (http://mdlottery.com/games/pick-3/payouts)

In that game, a successful bet of $\$ 1.00$ pays out $\$ 50.00$ for a net gain to the player of $\$ 49.00$.
Imagine that the state also offers a $\$ 1.00$ scratch-off lottery game with the following net gain distribution:
Scratch-Off Lottery

| Net Gain | Probability |
| :---: | :---: |
| $-\$ 1.00$ | 0.9600 |
| $\$ 9.00$ | 0.0389 |
| $\$ 99.00$ | 0.0010 |
| $\$ 999.00$ | 0.0001 |

If you had a friend who wanted to spend $\$ 1.00$ each day for several days on only 1 of these 2 lottery games, which game would you recommend? Explain.

Name $\qquad$ Date $\qquad$

## Lesson 15: Using Expected Values to Compare Strategies

## Exit Ticket

1. Your older sister asks you which of two summer job opportunities she should take. She likes them both. Job $A$ is self-employed; Job B works with a friend. The probability distributions for the amount of money that can be earned per day, $X$, follow. In Job B, money earned will be split evenly between your sister and her friend.

| Self-Employed <br> Job A |  |
| :---: | :---: |
| $X$ | Probability |
| 20 | 0.2 |
| 30 | 0.4 |
| 35 | 0.3 |
| 45 | 0.1 |


| With Friend <br> Job B |  |
| :---: | :---: |
| $X$ | Probability |
| 50 | 0.3 |
| 75 | 0.6 |
| 100 | 0.1 |

Which opportunity would you recommend that your sister pursue? Explain why in terms of expected value.
2. A carnival game consists of choosing to spin the following spinner once or roll a pair of fair number cubes once. If the spinner lands on 0 , you get no points; if it lands on 3 , you get 3 points; if it lands on 6 , you get 6 points. If the two number cubes sum to a prime number, you get 4 points. If the sum is not a prime number, you get 0 points. Should you spin the spinner or choose the number cubes if you want to maximize the expected number of points obtained? Explain why or why not. Note: The spinner is broken up into wedges representing $\frac{1}{2}, \frac{1}{3}$, and $\frac{1}{6}$.


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Date $\qquad$

## Lesson 16: Making Fair Decisions

## Exit Ticket

1. Both Carmen and her brother Michael want to borrow their father's car on a Friday night. To determine who gets to use the car, Carmen wants her father to roll a pair of fair dice. If the sum of the two dice is $2,3,11$, or 12 , Carmen gets to use the car. If the sum of the two dice is 4 or 10 , then Michael can use the car. If the sum is any other number, then the dice will be rolled again. Michael thinks that this is not a fair way to decide. Is he correct? Explain.
2. Due to a technology glitch, an airline has overbooked the number of passengers in economy class on a flight from New York City to Los Angeles. Currently, there are 150 passengers that have economy class tickets, but there are only 141 seats on the plane. There are two seats available in first class and one seat available in business class.
a. Explain how the ticket agent could use a random number generator to make a fair decision in moving some passengers to either the first or business class sections of the plane and to rebook the extra passengers to a later flight.
b. Is there any other way for the ticket agent to make a fair decision? Explain.

Name $\qquad$ Date $\qquad$

## Lesson 17: Fair Games

## Exit Ticket

A game is played with only the four kings and four jacks from a regular deck of playing cards. There are three "oneeyed" cards: the king of diamonds, the jack of hearts, and the jack of spades. Two cards are chosen at random without replacement from the eight cards. Each one-eyed card is worth $\$ 2.00$, and non-one-eyed cards are worth $\$ 0.00$. In the following table, JdKs indicates that the two cards chosen were the jack of diamonds and the king of spades. Note that there are 28 pairings. The one-eyed cards are highlighted.

| JcJd | JcJh | JcJs | JcKc | JcKd | JcKh | JcKs |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| JdJh | JdJs | JdKc | JdKd | JdKh | JdKs |  |
| JhJs | JhKc | JhKd | JhKh | JhKs |  |  |
| JsKc | JsKd | JsKh | JsKs |  |  |  |
| KcKd | KcKh | KcKs |  |  |  |  |
| KdKh | KdKs |  |  |  |  |  |
| KhKs |  |  |  |  |  |  |

a. What are the possible amounts you could win in this game? Write them in the cells of the table next to the corresponding outcome.
b. Find the the expected winnings per play.
c. How much should you be willing to pay per play of this game if it is to be a fair game?

Name $\qquad$ Date $\qquad$

## Lesson 18: Analyzing Decisions and Strategies Using Probability

## Exit Ticket

1. In a Home Décor store, $23 \%$ of the customers have reward cards. Of the customers who have reward cards, $68 \%$ use the self-checkout, and the remainder use the regular checkout. Of the customers who do not have reward cards, $60 \%$ use the self-checkout, and the remainder use the regular checkout.
a. Construct a hypothetical 1,000-customer two-way frequency table with columns corresponding to whether or not a customer uses the self-checkout and rows corresponding to whether or not a customer has a reward card.

|  | Self-Checkout | Regular Checkout | Total |
| :--- | :--- | :---: | :---: |
| Has Reward Card |  |  |  |
| Does Not Have Reward Card |  |  |  |
| Total |  |  | 1,000 |

b. What proportion of customers who use the self-checkout do not have reward cards?
c. What proportion of customers who use the regular checkout do not have reward cards?
d. If a researcher wishes to maximize the proportion of nonreward cardholders in a study, would she be better off selecting customers from those who use the self-checkout or the regular checkout? Explain your reasoning.
2. At the end of a math contest, each team must select two students to take part in the countdown round. As a math team coach, you decide to randomly select two students from your team. You would prefer that the two students selected consist of one girl and one boy. Would you prefer to select your two students from a team of 6 girls and 6 boys or a team of 5 girls and 5 boys? Show your calculations and explain how you reached your conclusion.

Name
Date $\qquad$

## Lesson 19: Analyzing Decisions and Strategies Using Probability

## Exit Ticket

1. A coffee machine has two nozzles. It is known that the amount of coffee dispensed by the first nozzle is approximately normally distributed with mean 7.1 oz . and standard deviation 0.41 oz . and that the amount of coffee dispensed by the second nozzle is approximately normally distributed with mean 7.2 oz . and standard deviation 0.33 oz . If a person is using an 8 oz . cup, which nozzle should he use to minimize the probability that the cup will be overfilled and the coffee will spill?
2. Ron's Joke Store offers a coin that is supposed to be weighted toward heads. Caitlin tries out one of these coins. She flips the coin three times and gets a head on all three flips.
a. If the coin were fair, what would the probability be that Caitlin would get heads on all three flips? (Round your answer to the nearest thousandth.)
b. Should Caitlin's result of heads on all three flips lead her to conclude that the coin is weighted toward heads? Explain.

## Appendix

## Standard Normal Curve Areas

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.8 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| -3.7 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| -3.6 | 0.0002 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| -3.5 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 |
| -3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| -3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| -3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| -3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| -3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| -2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| -2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| -2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| -2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| -2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| -2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| -2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| -2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| -2.1 | 0.0179 | 0.0174 | 0.0160 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| -2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| -1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| -1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| -1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| -1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| -1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0599 |
| -1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| -1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| -1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| -1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| -1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| -0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| -0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| -0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| -0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| -0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| -0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| -0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| -0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| -0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| -0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |


| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.7 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.8 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |

Name $\qquad$ Date $\qquad$

1. An assembler of computer routers and modems uses parts from three sources. Company A supplies $60 \%$ of the parts, company B supplies $30 \%$ of the parts, and company C supplies the remaining $10 \%$ of the parts. From past experience, the assembler knows that $3 \%$ of the parts supplied by company $A$ are defective, $5 \%$ of the parts supplied by company $B$ are defective, and $8 \%$ of the parts supplied by company C are defective. If a part is selected at random, what is the probability it is defective?
2. In the game of roulette, a wheel with different slots is spun. A ball is placed in the wheel and bounces around until it settles in one of the 38 slots, all of equal size. There are 18 red slots, 18 black slots, and 2 green slots. Suppose it costs $\$ 1.00$ to play one round of the game. A "color bet" allows you to pick either red or black; if the ball lands on your color, you win $\$ 2.00$. If the ball lands on either of the other two colors, you do not win any money.
a. In this game, for each spin you win either $\$ 2.00$ or $\$ 0.00$. Determine the probabilities of winning each amount.

b. Calculate the expected winnings in one spin of the wheel.
c. Are your expected winnings in one spin of the wheel larger or smaller than the $\$ 1.00$ bet?
d. A casino's profit is equal to the amount of bets minus the amount of winnings. For each spin of the Roulette wheel, the expected profit is $\$ 1.00$ minus your expected winnings. What is the casino's expected profit? Explain why the game of roulette is still attractive to a professional casino even though the expected profit to the casino is such a small amount on each spin.
e. The slots are also numbered $0,00,1-36$. If you bet on a number and win, you win $\$ 36.00$. Which bet is better for you: a number bet or a color bet? Explain your decision.
f. Suppose you plan to construct a wheel with 12 slots: 5 red, 5 black, and 2 green. You plan to pay $\$ 5.00$ in winnings if someone picks a color (red or black) and the ball lands on that color. How much should you charge someone to play (the bet amount) so that you have created a fair game?
3. In New York, the Mega Ball jackpot lottery asks you to pick 5 numbers (integers) from 1 to 59 (the "upper section") and then pick a Mega Ball number from 1 to 35 (the "lower section"). You win $\$ 10,000$ if you match exactly 4 of the 5 numbers from the upper section and match the Mega Ball number from the lower section. The winning number(s) for each section are chosen at random without replacement. Determine the probability of correctly choosing exactly 4 of the winning numbers and the Mega Ball number.
4. A blood bank is screening a large population of donations for a particular virus. Suppose $5 \%$ of the blood donations contain the virus. Suppose you randomly select 10 bags of donated blood to screen. You decide to take a small amount from each of the bags and pool these all together into one sample. If that sample shows signs of the virus, you then test all of the original 10 bags individually. If the combined sample does not contain the virus, then you are done after just the one test.
a. Determine the expected number of tests to screen 10 bags of donated blood using this strategy.
b. Does this appear to be an effective strategy? Explain how you know.
5. Twenty-five sixth-grade students entered a math contest consisting of 20 questions. The student who answered the greatest number of questions correctly will receive a graphing calculator. The rules of the contest state that if two or more students tie for the greatest number of correct answers, one of these students will be chosen to receive the calculator.

No student answered all 20 questions correctly, but four students (Allan, Beth, Carlos, and Denesha) each answered 19 questions correctly.

What would be a fair way to use two coins (a dime and a nickel) to decide which student should get the calculator? Explain what makes your method fair.
6. A cell phone company offers cell phone insurance for $\$ 7.00$ a month. If your phone breaks and you submit a claim, you must first pay a $\$ 200.00$ deductible before the cell phone company pays anything. Suppose the replacement cost for a phone is $\$ 650.00$. This means if you break your phone and have insurance, you have to pay only $\$ 200.00$ toward the replacement cost. This plan has a limit of two replacements; if you break your phone more than twice in one year, you pay for the full replacement cost for the additional replacements.

Suppose that within one year, there is a $48 \%$ chance that you do not break your phone, a $36 \%$ chance that you break it once, a $12 \%$ chance that you break it twice, a $3 \%$ chance that you break it three times, and a $1 \%$ chance that you break it four times.
a. Calculate the expected one-year cost of this insurance plan based on the monthly cost and the expected repair costs.
b. Determine your expected replacement costs if you do not purchase insurance.
c. Does this insurance plan seem to be a good deal? Explain why or why not.

