## Lesson 10: Determining Discrete Probability Distributions

## Classwork

## Exercises

Recall this example from Lesson 9:
A chance experiment consists of flipping a penny and a nickel at the same time. Consider the random variable of the number of heads observed.

The probability distribution for the number of heads observed is as follows.

| Number of Heads | 0 | 1 | 2 |
| :--- | :---: | :---: | :---: |
| Probability | 0.25 | 0.50 | 0.25 |

1. What is the probability of observing exactly 1 head when flipping a penny and a nickel?
2. Suppose you will flip two pennies instead of flipping a penny and a nickel. How will the probability distribution for the number of heads observed change?
3. Flip two pennies and record the number of heads observed. Repeat this chance experiment three more times for a total of four flips.
4. What proportion of the four flips resulted in exactly 1 head?
5. Is the proportion of the time you observed exactly 1 head in Exercise 4 the same as the probability of observing exactly 1 head when two coins are flipped (given in Exercise 1)?
6. Is the distribution of the number of heads observed in Exercise 3 the same as the actual probability distribution of the number of heads observed when two coins are flipped?
7. In Exercise 6, some students may have answered, "Yes, they are the same." But, many may have said, "No, they are different." Why might the distributions be different?

| Number of Heads | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- |
| Tally |  |  |  |

8. Combine your four observations from Exercise 3 with those of the rest of the class on the chart on the board. Complete the table below.
9. How well does the distribution in Exercise 8 estimate the actual probability distribution for the random variable number of heads observed when flipping two coins?

The probability of a possible value is the long-run proportion of the time that that value will occur. In the above scenario, after flipping two coins MANY times, the proportion of the time each possible number of heads is observed will be close to the probabilities in the probability distribution. This is an application of the law of large numbers, one of the fundamental concepts of statistics. The law says that the more times an event occurs, the closer the experimental outcomes naturally get to the theoretical outcomes.

A May 2000 Gallup Poll found that $38 \%$ of the people in a random sample of 1,012 adult Americans said that they believe in ghosts. Suppose that three adults will be randomly selected with replacement from the group that responded to this poll, and the number of adults (out of the three) who believe in ghosts will be observed.
10. Develop a discrete probability distribution for the number of adults in the sample who believe in ghosts.
11. Calculate the probability that at least one adult but at most two adults in the sample believe in ghosts. Interpret this probability in context.
12. Out of the three randomly selected adults, how many would you expect to believe in ghosts? Interpret this expected value in context.

## Lesson Summary

- To derive a discrete probability distribution, you must consider all possible outcomes of the chance experiment.
- The interpretation of probabilities from a probability distribution should mention that it is the long-run proportion of the time that the corresponding value will be observed.


## Problem Set

1. A high school basketball player makes $70 \%$ of the free-throws she attempts. Suppose she attempts seven freethrows during a game. The probability distribution for the number of made free-throws out of seven attempts is displayed below.

| Number of <br> Completed <br> Free-throws | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.00022 | 0.00357 | 0.02501 | 0.09725 | 0.22689 | 0.31765 | 0.24706 | 0.08235 |

a. What is the probability that she completes at least three free-throws? Interpret this probability in context.
b. What is the probability that she completes more than two but less than six free-throws? Interpret this probability in context.
c. How many free-throws will she complete, on average? Interpret this expected value in context.
2. In a certain county, $30 \%$ of the voters are Republicans. Suppose that four voters are randomly selected.
a. Develop the probability distribution for the random variable number of Republicans out of the four randomly selected voters.
b. What is the probability that no more than two voters out of the four randomly selected voters will be Republicans? Interpret this probability in context.
3. An archery target of diameter 122 cm has a bulls-eye with diameter 12.2 cm .
a. What is the probability that an arrow hitting the target hits the bulls-eye?
b. Develop the probability distribution for the random variable number of bulls-eyes out of three arrows shot.
c. What is the probability of an archer getting at least one bulls-eye? Interpret this probability in context.
d. On average, how many bulls-eyes should an archer expect out of three arrows? Interpret this expected value in context.

4. The probability that two people have the same birthday in a room of 20 people is about $41.1 \%$. It turns out that your math, science, and English classes all have 20 people in them.
a. Develop the probability distribution for the random variable number of pairs of people who share birthdays out of three classes.
b. What is the probability that one or more pairs of people share a birthday in your three classes? Interpret the probability in context.
5. You go to the warehouse of the computer company you work for because you need to send eight motherboards to a customer. You realize that someone has accidentally reshelved a pile of motherboards you had set aside as defective. Thirteen motherboards were set aside and 172 are known to be good. You're in a hurry, so you pick eight at random. The probability distribution for the number of defective motherboards is below.

| Number of <br> Defective <br> Motherboards $0^{2}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.5596 | 0.3370 | 0.0888 | 0.0134 | 0.0013 | 0.3177 | $7.6 \times 10^{-5}$ | $6.1 \times 10^{-8}$ | $5.76 \times 10^{-10}$ |

a. If more than one motherboard is defective, your company may lose the customer's business. What is the probability of that happening?
b. You are in a hurry and get nervous, so you pick eight motherboards, then second-guess yourself and put them back on the shelf. You then pick eight more. You do this a few times then decide it's time to bite the bullet and send eight motherboards to the customer. On average, how many defective motherboards are you choosing each time? Is it worth the risk of blindly picking motherboards?

