

Lesson 12: Estimating Probability Distribution Empirically

Classwork

Exploratory Challenge 1/Exercises 1–2: Moving Along

In a certain game, you toss 2 dice and find the difference of the numbers showing on the faces. You move along a number line according to the absolute value of the difference. For example, if you toss a 6 and a 3 on the first toss, then you move 3 spaces from your current position on the number line. You begin on the number 0, and the game ends when you move past 20 on the number line.

1. How many rolls would you expect it to take for you to get to 20? Explain how you would use simulation to answer this question.

2. Perform the simulation with your partner.
 - a. What is the expected value for the distance moved on 1 toss of 2 dice? Interpret your answer in terms of playing the game.

 - b. Use your expected value from part (a) to find the expected number of tosses that would put you past 20 on the number line.

Exploratory Challenge 2/Exercises 3–4: Lemon Flavor

Cough drops come in a roll with 2 different flavors, lemon and cherry. The same number of lemon and cherry cough drops are produced. Assume the cough drops are randomly packed with 30 per roll and that the flavor of a cough drop in the roll is independent of the flavor of the others.

3. Suppose you really liked the lemon flavor. How many cough drops would you expect to go through before finding 2 lemon cough drops in a row? Explain how you would use simulation to answer this question.

4. Carry out the simulation and use your data to estimate the average number of cough drops you would expect to go through before you found 2 lemon-flavored ones in a row. Explain what your answer indicates about 2 lemon-flavored cough drops in a row.

Lesson Summary

In this lesson, you learned that

- You can estimate probability distributions for discrete random variables using data from simulating experiments.
- Probabilities from an estimated probability distribution for a discrete random variable can be interpreted in terms of long-run behavior of the random variable.
- An expected value can be calculated from an estimated probability distribution and interpreted as a long-run average.

Problem Set

1. Suppose the rules of the game in Exploratory Challenge 1 changed.
If you got an absolute difference of
 - 3 or more, you move forward a distance of 1;
 - 1 or 2, you move forward a distance of 2;
 - 0, you do not move forward.
 - a. Use your results from Exploratory Challenge 1/Exercise 2 to estimate the probabilities for the distance moved on 1 toss of 2 dice in the new game.
 - b. Which distance moved is most likely?
 - c. Find the expected value for distance moved if you tossed 2 dice 10 times.
 - d. If you tossed the dice 20 times, where would you expect to be on the number line, on average?
2. Suppose you were playing the game of Monopoly, and you got the Go to Jail card. You cannot get out of jail until you toss a double (the same number on both dice when 2 dice are tossed) or pay a fine.
 - a. If the random variable is the number of tosses you make before you get a double, what are possible values for the random variable?
 - b. Create an estimated probability distribution for how many times you would have to toss a pair of dice to get out of jail by tossing a double. (You may toss actual dice or use technology to simulate tossing dice.)
 - c. What is the expected number of tosses of the dice before you would get out of jail with a double?

3. The shuttle company described in the Exit Ticket found that when they make 11 reservations, the average number of people denied a seat per shuttle is about 1 passenger per trip, which leads to unhappy customers. The manager suggests they take reservations for only 10 seats. But his boss says that might leave too many empty seats.
- Simulate 50 trips with 10 reservations, given that in the long run, 10% of those who make reservations do not show up. (You might let the number 1 represent a no-show and a 0 represent someone who does show up. Generate 10 random numbers from the set that contains one 1 and nine 0s to represent the 10 reservations and then count the number of 1s.)
 - If 3 people do not show up for their reservation, how many seats are empty? Explain your reasoning.
 - Use the number of empty seats as your random variable and create an estimated probability distribution for the number of empty seats.
 - What is the expected value for the estimated probability distribution? Interpret your answer from the perspective of the shuttle company.
 - How many reservations do you think the shuttle company should accept and why?