

Lesson 15: Using Expected Values to Compare Strategies

Classwork

Example 1

A math club has been conducting an annual fundraiser for many years that involves selling products. The club advisors have kept records of revenue for the products that they have made and sold over the years and have constructed the following probability distributions for the three most popular products. (Revenue has been rounded to the nearest hundred dollars.)

Candy		Magazine Subscriptions		Wrapping Paper	
Revenue	Probability	Revenue	Probability	Revenue	Probability
\$100.00	0.10				
\$200.00	0.10	\$200.00	0.4		
\$300.00	0.25	\$300.00	0.4	\$300.00	1.0
\$400.00	0.45	\$400.00	0.2		
\$500.00	0.05				
\$600.00	0.05				

Exercises 1–2

- The club advisors only want to offer one product this year. They have decided to let the club members choose which product to offer and have shared the records from past years. Assuming that these probability distributions were to hold for the coming fundraiser, which product should the club members recommend they sell? Explain.

2. The club advisors forgot to include overhead costs with the past revenue data. The overhead costs are \$80.00 for candy, \$20.00 for magazine subscriptions, and \$40.00 for wrapping paper. Will this additional information change the product that the math club members recommend they sell? Why?

Example 2

A game on television has the following rules. There are four identical boxes on a table. One box contains \$1.00; the second, \$15.00; the third, \$15,000.00, and the fourth, \$40,000.00. You are offered the choice between taking \$5,000 or taking the amount of money in one of the boxes you choose at random.

Exercises 3–4

3. Suppose that you want to buy a \$7,500.00 pre-owned car. What should you do? Take the \$5,000.00 or choose a box? Why?

4. What should you do if you want to buy a \$20,000.00 brand-new car? Take the \$5,000.00 or choose a box? Why?

Example 3

In a certain two-player game, players accumulate points. One point is earned for a win, half a point is earned for a tie, and zero points are earned for a loss. A match consists of two games. There are two different approaches for how the game can be played—boldly (B) or conservatively (C). Before a match, Henry wants to determine whether to play both games boldly (BB), one game boldly and one game conservatively (BC or CB), or both games conservatively (CC). Based on years of experience, he has determined the following probabilities for a win, a tie, or a loss depending on whether he plays boldly or conservatively.

Approach	Win (W)	Tie (T)	Lose (L)
Bold (B)	0.45	0.0	0.55
Conservative (C)	0.0	0.8	0.2

How should Henry play to maximize the expected number of points earned in the match? The following exercises will help you answer this question.

Exercises 5–14

5. What are the possible values for the total points Henry can earn in a match? For example, he can earn $1\frac{1}{2}$ points by WT or TW (he wins the first game and ties the second game or ties the first game and wins the second game). What are the other possible values?

6. If Henry plays both games boldly (BB), find the probability that Henry will earn
- a. 2 points
 - b. $1\frac{1}{2}$ points
 - c. 1 point
 - d. $\frac{1}{2}$ point
 - e. 0 points
7. What is the expected number of points that Henry will earn if he plays using a BB strategy?
8. If Henry plays both games conservatively (CC), find the probability that Henry will earn
- a. 2 points
 - b. $1\frac{1}{2}$ points

- c. 1 point
- d. $\frac{1}{2}$ point
- e. 0 points
9. What is the expected number of points that Henry will earn if he plays using a CC strategy?
10. If Henry plays the first game boldly and the second game conservatively (BC), find the probability that Henry will earn
- a. 2 points
- b. $1\frac{1}{2}$ points
- c. 1 point
- d. $\frac{1}{2}$ point

e. 0 points

11. What is the expected number of points that Henry will earn if he plays using a BC strategy?

12. If Henry plays the first game conservatively and the second game boldly (CB), find the probability that Henry will earn

a. 2 points

b. $1\frac{1}{2}$ points

c. 1 point

d. $\frac{1}{2}$ point

e. 0 points

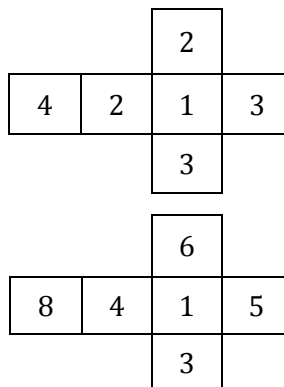
13. What is the expected number of points that Henry will earn if he plays using a CB strategy?
14. Of the four possible strategies, which should Henry play in order to maximize his expected number of points earned in a match?

Lesson Summary

- Making decisions in the face of uncertainty is one of the primary uses of statistics.
- Expected value can be used as one way to decide which of two or more alternatives is best for either maximizing or minimizing an objective.

Problem Set

1. A game allows you to choose what number cubes you would like to use to play. One pair of number cubes is a regular pair in which the sides of each cube are numbered from 1 to 6. The other pair consists of two different cubes as shown below. For all of these number cubes, it is equally likely that the cube will land on any one of its six sides.



- a. Suppose that you want to maximize the expected sum per roll in the long run. Which pair of number cubes should you use? Explain why.
 - b. Imagine that you are playing a game in which you earn special privileges by rolling doubles (i.e., the same number on both cubes). Which number cubes would you prefer to use? Explain.
2. Amy is a wedding planner. Some of her clients care about whether the wedding is held indoors or outdoors depending on weather conditions as well as respective costs. Over the years, Amy has compiled the following data for June weddings. (Costs are in thousands of dollars.)

Weather	Cost Indoors	Cost Outdoors	Probability
Cold and sunny	\$29	\$33	0.15
Cold and rainy	\$30	\$40	0.05
Warm and sunny	\$22	\$27	0.45
Warm and rainy	\$24	\$30	0.35

- a. What is the expected cost of a June wedding held indoors?
- b. What is the expected cost of a June wedding held outdoors?
- c. A new client has her heart set on an outdoor wedding. She has at most \$25,000.00 available. What do you think Amy told the client and why?

3. A venture capitalist is considering two investment proposals. One proposal involves investing \$100,000.00 in a green alternative energy source. The probability that it will succeed is only 0.05, but the gain on investment would be \$2,500,000.00. The other proposal involves investing \$300,000.00 in an existing textile company. The probability that it will succeed is 0.5 and the gain on investment would be \$725,000.00. In which proposal should the venture capitalist invest? Explain.
4. A student is required to purchase injury insurance in order to participate on his high school football team. The insurance will cover all expenses incurred if the student is injured during a football practice or game, but the student must pay a deductible for submitting a claim. There is also an up-front cost to purchase the injury insurance.
- Plan A costs \$75.00 up front. If the student is injured and files a claim, the deductible is \$100.00.
 - Plan B costs \$100.00 up front. If the student is injured and files a claim, the deductible is \$50.00.

Suppose there is a 1 in 5 chance of the student making a claim on the insurance policy. Which plan should the student choose? Explain.